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EARLY ELECTRICAL DISCOVERIES BY BENJAMIN FRANKLIN AND HIS CONTEMPORARIES.

BY

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When Benjamin Franklin drew the lightning from the clouds and identified this with the then little understood medium called "electricity," he initiated a movement that has led to untold blessings to mankind. The significance of this discovery was not so much in its intrinsic importance but rather in the unique personality of the discoverer and in the enthusiasm with which he carried on his studies in a field that up to that time had received little consideration from the scientific world.

In the light of our present knowledge many of Franklin's experiments seem quite trivial in character, and they impressed some of his friends in the same way. The wide range of his activities is illustrated by the interest he showed in early experiments in aviation. When questioned as to any possible use that could be derived from flying a balloon, he replied "what is the use of a new-born baby." He evidently had the same faith in the future development of the electrical industry, which was his major scientific interest, and possibly he made the same reply to those who questioned his interest in this field for in the later years of his life he was frequently called the father of electricity. ("Life and Writings of Benjamin Franklin," by A. H. Smyth, V. I, p. 77.)

Benjamin Franklin was born in Boston on January 17, 1706, into a family of 17 children, 7 of these being children of his father's first wife. Benjamin was the youngest son in this large group and he had the unusual distinction of completing a direct succession of youngest sons through five generations. He received only two years of formal school

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instruction and at the age of ten years was taken home to assist in his father's business which was that of a tallow-chandler and soap-boiler.

Here he was employed in cutting wicks, filling candle molds and acting as general errand boy for the establishment. Living in the port of Boston where ships were arriving from all parts of the world, and being galled by the confining nature of this menial work, he was strongly tempted to run away from home for a seafaring life. He was saved from this misfortune however by a new interest through the arrival of his brother James from England with printing press and equipment that enabled him to build up a business as printer and publisher. Benjamin was apprenticed to his brother and served with him until he was seventeen years old. Here he became an expert type setter and began his career as a writer, though he later confessed that his early attempts "were wretched stuff in the Grub-street ballad style."

After having many disputes with his brother he severed his relations with him and wishing to be entirely free from his brother's influences, he took ship for New York. Then after crossing New Jersey, largely on foot, he arrived in Philadelphia in October 1723. He came without any knowledge of the town and with only a few shillings in his pocket but having his trade as a profession he soon found employment. His genius for making friends soon became apparent and within a year after his arrival he had gained a large circle of friends. Among others was Sir William Keith, the governor of the province, who took him under his patronage. It was at his suggestion that Franklin made his first visit to London where he remained for two years.

Upon his return to Philadelphia he again took up his profession as publisher. Through his writings and by his industry and good sense he became the recognized head of journalism in the colonies. In 1731 he established the first circulating library on the continent; the next year he started his Poor Richard Almanac which was continued for twenty-five years and which attained great popularity and wide circulation.

In 1736 Franklin began his political career by becoming a member of the General Assembly. At about this time he organized the first police force and fire department in the colonies and provided for having the principal streets in Philadelphia paved. A few years later he was active in movements for starting a hospital, for founding the American Philosophical Society and the establishment of an academy which later became the University of Pennsylvania. It was not until 1746 that he first became interested in the study of electricity. He described how this occurred in the following words:

> In 1746 being in Boston, I met there with a Dr. Spence, who had lately arrived from Scotland, and he showed me some electric experiments. They were imperfectly performed, as he was not very expert; but being a subject quite new to me, they equally surprised and pleased me. Soon after my return to Philadelphia, our library company received from Mr.

Peter Collinson, Fellow of the Royal Society of London, a present of a glass tube, with some account of the use of it in making such experiments. I eagerly seized the opportunity of repeating what I had seen in Boston and by much practice acquired great readiness in performing those, also, which we had an account from England, adding a number of new ones. I say with much practice for my house was continually full, for some time, with persons who came to see these new wonders.

To divide a little this incumbrance among my friends I caused a number of similar tubes to be blown in our glass-house, with which they furnished themselves, so that at length we had several performances. Among these, the principal was Mr. Kinnersley, an ingenious neighbor, who being out of business, I encouraged him to undertake showing the experiments for money, and drew up for him two lectures, in which the experiments were arranged in such order, and accompanied with explanations in such a method, as that the foregoing should assist in comprehending the following. He procured an elegant apparatus for the purpose, in which all the little machines that I had roughly made for myself were neatly formed by instrument makers. His lectures were well attended, and gave great satisfaction; and after some time he went through the colonies, exhibiting them in every capital town, and picked up some money. ("Benjamin Franklin, His Autobiography," p. 244. Edited by H. Haslings Weld, New York, 1855.)

As has been noted above it was not until Franklin's day that scientists began to take a serious interest in electrical phenomena so that it would seem well to consider briefly what electrical discoveries had been made up to that time. It had long been known that certain bodies, later called nonconductors, could receive an electrical charge through friction. This charge could be detected either by contact with one's body when a prickling sensation was produced or by bringing the charge into the presence of shreds of silk, paper or other light material, when these would first be attracted to the surface of the charged body and after coming in contact with that surface they would be repelled from it. It was also known that these light objects when charged from different sources behaved in an uncertain manner when brought near to each other, in some cases there was an attraction between them and in others a repulsion. When they received their charge from the same source they were always repelled. This discovery led the development of the first electrical instrument which was called an *electroscope*. It took various forms but one of the best known consisted of two strips of gold leaf inclosed in a glass bottle. They were suspended from a common metal terminal which extended through the neck of the bottle so that when a charged body was brought into contact with the terminal its presence was indicated by a separation of the gold leaves.

In 1729 Stephen Gray, a member of the Royal Society of England, made experiments to determine the difference in the behavior of conductors and nonconductors for transmitting an electric charge. He used lines of silk, iron and brass. He gradually extended the length of his lines up to about 900 feet. He employed a brass leaf electroscope as a detector. (*Philosophical Transactions*, Vol. 37, p. 18, 1731.)

At about this time Charles Francois de Cisternay du Fay, a member of the Royal Academy of France, developed his two fluid theory of electricity. He had noticed the capricious action of charged bodies

noted above, namely that in some cases such bodies were attracted to each other while in other cases they were repelled. He explained this on the principle that there were two kinds of electricity, one of which he called *vitreous electricity* which could be produced by friction upon glass, rock-crystal, precious stones, hair of animals, wool and many other bodies. The other he called *resinous electricity*, produced from amber, copal, gum-lack, silk, thread, paper and a vast number of other substances. (*Philosophical Transactions*, Vol. 38, p. 258, 1734.)

In 1745 the *Leyden jar* or phial was developed, so named through the work of Pieter van Musschenbroek, Professor of Philosophy and Mathematics in the University of Leyden. He had observed that electrified bodies soon lost their charge when exposed to the air and conceived the idea that more stable conditions could be obtained if he could devise means for enclosing the charge in a nonconductor. He partly filled a glass bottle with water while his assistant held this in his hand, then during the manipulation of the circuit when it was being charged the assistant received such a shock in his hand and arm that he clearly demonstrated the effectiveness of the device. William Watson, a member of the Royal Society of London, brought the jar into its modern form by coating both the inside and outside of the jar with metal. (*Memoires de l'Academie Royale des Sciences*, Paris, for 1746, p. 1).

The development of the Leyden jar provided means for producing much stronger electrical effects than had previously been possible. Also more powerful generators were designed so the way was paved for a considerable advance in the study of electricity. This was reflected in a great increase in the number of papers recounting experiments in electricity that were appearing in the technical papers, especially in England and France. Martin Folkes, President of the Royal Society of London, commenting on this situation made the following statement: "Electricity seems to furnish an inexhaustible fund for inquiry and such phenomena so various and so wonderful, can arise only from causes very great and extensive, and such as must have been designed by the Almighty author of nature for the production of very great effects." (*Philosophical Transactions*, Vol. 44, Part I, p. 41, 1746.)

William Watson in England and others on the continent of Europe found that light spirits could be ignited by an electrical discharge and that visible sparks could be seen that were accompanied by sounds resembling a miniature thunder storm. (*Philosophical Transactions*, Vol. 43, p. 481, 1745.) Watson then started a study of the electrical conductivity of different substances that was continued for several years. He pointed out the difference in the behavior of conductors and nonconductors and showed that metals were best for conducting a charge. In 1747 he made a true electrical transmission experiment. He laid a wire across Westminster Bridge in London and stationed an

observer on each bank of the river. The observer on the Westminster side held one terminal of the wire in one hand and made contact with the river by inserting an iron rod which he held in his other hand into the water. The observer on the Surry shore took hold of the wire with one hand and grasped with his other a large phial filled with iron filings and coated with sheet lead highly electrified from a glass globe installed in a neighboring house. A third observer standing near the second dipped an iron rod which he held in one hand into the water while he touched with his other hand the iron hook of the phial which made contact with the iron filings. As soon as this contact was made the charge snapped and was felt by each of the three observers. This experiment was repeated several times with equal success but during the progress of the tests a great crowd of spectators had collected on the bridge which caused considerable annoyance by breaking the transmission wire and in other ways interrupting the experiment. The total length of this circuit including the water was about 2,400 feet.

Then Watson decided to continue these tests but in order to avoid interference from a curious audience he moved his equipment into the open country. He varied his experiment by using an overhead wire for his outgoing circuit and a bare wire laid on the ground for the return circuit. Starting with a total length of line of about one mile, he gradually increased this to nine miles and received signals successfully in each case. He then made an attempt to determine the velocity of the electrical transmission. A gun was fired at the far end of the line at the moment a charge from a Leyden jar was introduced into the circuit. The difference in the time between the reception of the charge and that of the sound of the gun was noted and knowing the velocity of sound it was estimated that the velocity of transmission of the electrical charge was practically instantaneous compared with that of sound. (*Philosophical Transactions*, Vol. 45, p. 49, 1747.)

While Watson was conducting these transmission experiments he was making many others in order to obtain more specific information regarding the properties of electricity, and he concluded that what we call *electricity* is a very subtile and elastic fluid common to all bodies which in their natural state have the same electrical density. That this fluid manifests itself only when bodies capable of receiving more than their natural share are operated upon by certain well known methods and that then they show their electrical condition by attracting or repelling light objects, by a snapping sound, by sparks and in fact may be capable of affecting any of our senses. Also that no snapping is observed when two bodies have the same density. That the intensity of the snapping is determined by the difference in the density of the two objects, and that certain bodies which he called electrics *per se* have the property of transmitting electricity. (*Philosophical Transactions*, Vol. 45, p. 95, 1747.)

Within a year from the time that Franklin began to be interested in the study of electricity he received his first recognition as a scientist from Europe. The first reference to his scientific work to appear in the *Philosophical Transactions* was given by William Watson in the paper that has just been considered. He stated that Mr. Collinson, a member of the Royal Society, had recently received a paper concerning electricity from an ingenious gentleman, Mr. Franklin, a friend of his in Pennsylvania. The paper was dated June 1, 1747. By a curious coincidence which sometimes occurs in scientific investigations, two students entirely without knowledge of the other's work, covered much the same ground but did not always reach the same conclusions. In this case while Watson was carrying on his research in London, as outlined above, Franklin was making his experiments in Philadelphia. He gave his results in the paper which he sent to Mr. Collinson and which was presented to the Royal Society. Franklin designated the two electrical states that a body might receive as due to a positive or negative charge, terms that have continued in use to the present day. (*Philosophical Transactions*, Vol. 45, p. 98, 1747.)

In July 1747 Franklin investigated the effect of different forms of terminals upon the discharge of a Leyden jar. He used for one terminal a cast iron shot about 4 inches in diameter, when he approached this with the other terminal which consisted of a sharply pointed bodkin the discharge took place through an air gap of 6 to 8 inches, but if he substituted for the bodkin a blunt body, the terminals could be brought to one inch of each other before the discharge took place. ("Works of Benjamin Franklin," by Jared Sparks, Boston, 1840, Vol 5, p. 181.) This experiment guided Franklin later in the design of the terminals for his lightning rods.

It was at about this time that Franklin began his study of lightning with the definite purpose of identifying it with electricity. Other scientists had noted that the vivid flash accompanied by a crackling sound which occurred on the discharge of a group of highly charged Leyden jars was quite suggestive of the lightning flash followed by thunder. It was at about this time that Abbé Jean Antoine Nollet, professor of physics at the University of Paris and member of the French Academy, a prolific writer on electrical matters, expressed the hope that some one would attempt to prove that the thunder in the hands of nature is what electricity is in ours and that the wonders which we now exhibit at our pleasure are little imitations of those effects that frighten us.

In the spring of 1748 Franklin wrote to Peter Collinson describing experiments he had made on charging and discharging Leyden jars and in studying the effect of grouping them in different combinations. But he appeared to be rather discouraged at the futility of this work for he closed his letter with the following paragraph:

Chagrined a little that we have hitherto been able to produce nothing in the way of use to mankind and hot weather coming on, when electrical experiments are not so agreeable, 'tis proposed to put an end to them for the season, somewhat humorously in a party of pleasure on the banks of the Skuylkil. Spirits at the same time are to be fired by a spark sent from side to side through the river, without any other conductor than the water—an experiment which we some time since performed, to the amazement of many. A turkey is to be killed for our dinner by the electric shock, and roasted by the electric jack, before a fire kindled by the electrified bottle, when the healths of all the famous electricians in England, Holland, France and Germany are to be drank in electrified bumpers, under the discharge of guns from the electrified battery. (Spark's "Franklin," Vol. 5, p. 210.)

Answering a letter from this same English friend he wrote later in 1748 expressing his pleasure in hearing that the paper which he had presented the previous year had met with the approval of the Royal Society and also that his ingenious friend Mr. Watson was preparing another paper on the same subject. He stated that he himself had lately been doing very little experimental work in electricity but that he hoped to take it up again during the following winter, "as the approaching peace gives us a prospect of being more easy in our minds." ("Writings of Benjamin Franklin," by A. H. Smyth, Macmillan & Co., 1905, Vol 2, p. 364.)

Franklin's mention of hostilities has reference to an early stage in the French and Indian war with the colonies, and while his hope for an early peace was not realized he did find time to again take up his electrical studies. In the summers of 1749 and 1750 he wrote to his English friend, Peter Collinson, giving him many examples of the way in which lightning produced effects similar to those from a heavily charged Leyden jar. He also suggested a theory to explain how clouds as they collected before a storm might draw electricity from the earth. He confessed that he would not place too much reliance upon this theory and then made the following comments:

It is not of much importance to us to know the manner in which nature exerts her laws; 'tis enough if we know the laws themselves, 'tis of real use to know that china left in the air unsupported will fall and break; but *how* it comes to fall and why it breaks are matters of speculation; 'tis a pleasure indeed to know them, but we can preserve china without it. Thus in the present case to know the power of points in assisting in the discharge of Leyden jars may be of some use to mankind in preserving houses, churches, ships &c. from the strokes of lightning by directing us to fix on the highest parts of those edifices, upright rods of iron made sharp as a needle and gilt to prevent rusting and from the foot of those rods, a wire down the outside of the building into the ground, or down around one of the shrouds of a ship and down her side till it reaches the water. ("Writings of Benjamin Franklin," by A. H. Smyth, Vol. 2, pp. 411, 427 and 437.)

This appears to be the first time that Franklin had given an account of his famous lightning rod which was soon applied very widely not only throughout the colonies but also in Europe and since that time has been of inestimable value in the protection of property.

In the same paper quoted above Franklin described an experiment for proving definitely whether lighting was an electrical discharge.

This later became known as the *Philadelphia experiment* and it created a real sensation in the scientific world of that day, so Franklin's description of it is given in full:

> To determine whether the clouds that contain lightning are electrified or not, I propose an experiment to be tried where it may be done conveniently. On the top of some high tower or steeple place a kind of sentry-box big enough to contain a man and an electrical stand. From the middle of the stand let an iron rod rise and pass bending out of the door and then upright for 20 to 30 feet, pointed very sharp at the end. If the electric stand be kept clean and dry a man standing on it, when such clouds are passing low, might be electrified and afford sparks, the rod drawing fire to him from a cloud. If any danger to the man should be apprehended (though I think there should be none), let him stand on the floor of his box and then bring near to the rod the loop of a wire that has one end fastened to the leads, he holding it by a wax handle, so the sparks if the rod is electrified, will strike from the rod to the wire and not affect him. ("Writings of Benjamin Franklin," by Smyth, Vol. 2, p. 437.)

When Collinson showed Franklin's letter to other members of the Royal Society they ridiculed his theory and said that it was not worthy of being published in their *Transactions*. However, in the following year a short and very guarded note by William Watson was published in the *Transactions* in which Watson said that Franklin had a very thorough knowledge of electrical matters, that he had the head to conceive and the hands to execute his experiments, but that he held some few opinions with which Watson could not agree. (*Philosophical Transactions*, Vol. 47, p. 210, 1751.)

In the summer of 1752 Abbé Nollet saw a translation of Franklin's paper describing his Philadelphia experiments. He could not at first believe that such a work had come out of America, and said it must have been fabricated by his enemies in Paris to discredit him for the theory which he had developed regarding the nature of electricity. Afterwards, when he had been assured that there really existed such a person as Franklin in Philadelphia, he wrote a long letter to Franklin defending his own theory and denying the validity of Franklin's experiments and the conclusions deduced from them (Sparks, Vol. 5, p. 493) (*Philosophical Transactions*, Vol. 47, p. 553).

At first Franklin thought that he would answer this attack, but then he decided to let his papers shift for themselves believing it was better to spend any time that he could spare from his public business in making new experiments rather than disputing about those already made. He found that he was fully justified in this decision for during the summer of 1752, other scientists throughout Europe repeated Franklin's experiments and proved the validity of his theories which then became universally accepted. ("Autobiography of Benjamin Franklin," edited by John Bigelow, Lippincott & Co., 1868, p. 332.) De le Monnier repeated his experiments in Paris using an iron rod connected to the earth through an iron wire. He found that it was not necessary to provide a sharp point on the rod and that this might even be placed horizontally. A man thus electrified could set fire to spirits

of wine with his finger and repeat almost all of the useful experiments of artificial electricity. (*Philosophical Transactions*, Vol. 47, p. 557.) Messrs. LeRoy and Dalibard showed the Philadelphia experiments in Paris before the king and court and all the curious in the city flocked to see them. (*Philosophical Transactions*, Vol. 47, p. 557.)

Professor Ludolf and Mr. Mylius conducted experiments in Berlin using a pointed iron rod 12 feet long, raised on a 50 foot pole and connected to a summer house through a long iron wire. When a thunder storm approached great sparks appeared which could sometimes produce as violent shocks through the body as were obtained from Leyden jars. (*Philosophical Transactions*, Vol. 47, p. 559.) Similar experiments were also made at Florence, Bologna and other places on the continent.

In October 1752 Franklin wrote to his friend Peter Collinson as follows:

As frequent mention is made in the public papers from Europe of the success of the Philadelphia experiment for drawing electric fire from the clouds by means of pointed rods of iron erected on high buildings &c. it may be agreeable to the curious to be informed that the same experiment has succeeded in Philadelphia, though made in a different and more easy manner which anyone may try as follows:—Make a small cross of two light strips of cedar; the arms so long as to reach to the four corners of a larg thin silk handerchief when extended. Tie the corners of this to the extremities of the cross, so you have the body of the kite; which being properly accomodated with a tail, loop and string, will rise in the air like those made of paper; but this being a silk is fitter to bear wet and wind of a thunder storm without tearing. To the top of the upright stick of the cross is to be fixed a very sharp pointed wire rising a foot or more above the wood. To the end of the twine next to the hand is to tied a silk ribbon; and where the twine and silk join a key may be fastened.

The kite is to be raised when a thunder gust appears to be coming on and the person who holds the string must stand within a door or window, or under some cover so that the silk ribbon may not be wet; and care must be taken that the twine does not touch the frame of the door or window.

As soon as any of the thunder clouds come over the kite, the pointed wire will draw the electric fire from them and the kite, with all the twine will be electrified; and the loose filaments of the twine will stand out every way and be attracted by an approaching finger.

When the rain has wet the kite and twine, so that it can conduct the electric fire freely, you will find it will stream out plentifully from the key on the approach of your knuckle. At this key a phial may be charged; and from the electric fire thus obtained spirits may be kindled and all other electrical experiments be performed which are usually done by the help of a rubbed glass globe or tube and I think that the sameness of the electric matter and that of lightning has been completely demonstrated.

I was pleased to hear of the success of my experiments in France, and that they have begun to erect points on their buildings. We have placed them on our academy and state house spires. (*Philosophical Transactions*, Vol. 47, p. 565.)

In view of the tremendous havoc wrought by lightning, it may seem surprising that only one case of serious accident to any of those engaged in these experiments, has been recorded. George William Richman, professor of experimental philosophy and member of the Imperial Academy of Science in St. Petersburg, became interested in the study of lightning, repeating Franklin's Philadelphia experiment. He con-

structed a new form of electroscope that was provided with a graduated scale which he called a *demonstrator* of electricity. He erected several long iron bars on the roof of his house and connected these by chains to the electroscope in his laboratory. With this arrangement he had been observing the state of the atmosphere and also the magnitude of the charge he was able to obtain from his static generator. Upon the approach of a thunder storm in the summer of 1753, he brought a friend to his laboratory to show him the way in which his instrument would indicate the presence of electricity during a storm. When they entered the room the electroscope registered 4°, which the professor said meant that the storm was at a distance, but he said that if the indicator should rise to 45° or more, it would represent a truly dangerous condition. As he stood near the instrument a stroke of lightning, which it was said was of unusual intensity, hit the circuit. He was instantly killed, his friend was stunned and much wood work in the house was shattered in the path of the lightning as it went to the ground. (*Philosophical Transactions*, Vol. 49, pt. I, p. 61, 1755.)

Possibly it was by mere good fortune that no direct lightning strokes occurred during any of the other experiments that have been mentioned above. On the other hand Franklin himself met with an accident which might readily have ended his career when he was conducting quite a different kind of experiment. He gave the following account of this affair:

> I have lately made an experiment in electricity that I desire never to repeat. Two nights ago, being about to kill a turkey by the shock from two glass jars, containing as much electrical fire as forty common phials, I inadvertently took the whole through my arms and body, by receiving the fire from the united top wires with one hand while the other held a chain connecting with the outsides of both jars. The company present say, that the flash was very great and the crack as loud as a pistol; yet, my senses being instantly gone, I neither saw the one or heard the other; nor did I feel the stroke on my hand, though I afterwards found it raised a round swelling where the fire entered, as big as half a pistol bullet; by which you may judge of the quickness of the electric fire, which by this instance seems to be greater than that of sound, light or animal sensation. . . . You may communicate this to Mr. Bowdoin, as a caution to him, but do not make it more public for I am ashamed to have been guilty of so notorious a blunder; a match for that of the Irishman, whom my sister told me of, who, to divert his wife poured the bottle of gunpowder on a live coal; or of that other, who, being about to steal powder, made a hole in the cask with a hot iron. (Sparks, Vol. 5, p. 255.)

Toward the close of the year 1752 after various reports had been received from many parts of Europe in confirmation of Franklin's theory of lightning, it was noted that no experiments of this character were being reported from England. William Watson gave an explanation of this to the Royal Society as follows:

> After the communications which we have received from several of our correspondents in different parts of the continent acquainting us with the success of their experiments last summer in endeavor to extract the electric power from the atmosphere during a thunder storm in consequence of Mr. Franklin's hypothesis, it may be thought extraordinary that no accounts

have been laid before you of our success here from the same experiments. That no want of attention therefore may be attributed to those here, who have been hitherto conversant in these inquiries, I thought proper to appraise you, that though several members of the Royal Society as well as myself, did upon the first advices from France prepare and set up the necessary apparatus for this purpose, we were defeated in our expectations from the uncommon coolness and dampness of the air here during the whole summer. We had only one thunder storm in London and then the thunder was accompanied with rain, so that by wetting the apparatus the electricity was dissipated too soon to be perceived upon touching those parts of the apparatus which served to conduct it. This I say in general prevented us from verifying Mr. Franklin's hypothesis. (*Philosophical Transactions*, Vol. 47, p. 567.)

Experiments with electric kites continued to be made well into the nineteenth century. In 1834 William Sturgeon, who was celebrated for his discovery of the electromagnet, thought that sufficient interest was being shown in this subject to make it desirable for him to write a paper cautioning the public against the improper use of the electric kite. The following extracts have been taken from this paper:

Last Friday in the afternoon clouds began to form in various quarters of the heavens in rapid succession with every appearance of being highly electrical. I repaired to the Artillery grounds with an electrical kite and in a short time got it afloat, letting out string through the hand from a coil that was thrown on the ground. When about one hundred yards of the string had been let out, a tremendous discharge took place, which gave me such a blow in the chest and legs that I became completely stunned, let go the string and consequently the kite soon fell.

The accident was owing entirely to my neglect and it could not possibly have happened had I taken the following precaution. Let all the string intended to be employed be first taken off the coil and stretched on the ground. Let now the insulating cord, glass or whatever is used for the purpose, be attached to the kite string and fastened to a peg in the ground. Next fasten the kite to the other end of the string and let it ascend from the hand.

I find it convenient to have a sliding copper wire on the silken cord which can be moved by means of a long glass rod to any required distance from the main wired string, the other end being stuck in the ground. If the electric fire strikes two inches over the dry silken cord (and it will sometimes strike a yard), it would not be safe to approach it and no man could hold the string when it strikes over one inch.

The only method of getting the kite down during a storm with safety is to unfasten the silken cord from its hold and let all go so that the kite falls. (*Phil. Mag.*, V. 5, p. 317, 1834.)

The main wired string mentioned above evidently means that a special kite string had been devised having a fine wire central core in order to increase its conductivity.

William Sturgeon also gave a vivid account of the results that might be obtained through this ingenious experiment that had been devised by Franklin:

On June 14th, at about eight o'clock in the evening, an electric storm passed over this place exhibiting lightning the most splendid ever beheld. The rain fell so heavily that it was with some difficulty that I got the kite afloat; and when up its greatest altitude, I imagine, did not exceed fifty yards. The silken cord also, which had been intended for the insulation, soon became so completely wet that it was no insulator at all. Notwithstanding all these impediments being in the way, I was much gratified with the display of the electric matter issuing from the end of the string to the wire, one end of which was laid on the ground, and the other attached to the silk at about four inches distance from the reel of the kite string. An

uninterrupted play of fluid was seen over four inches of wet silken cord, not in sparks, but in a bundle of quivering purple ramifications, producing a noise similar to that produced by springing a watchman's rattle. Very long sparks, however, were frequently seen between the lower end of the wire (which rested on the grass) and the ground; several parts of the string towards the kite, where the wire was broken, were occasionally beautifully illuminated. The noise from the string in the air was like to the hissing of an immense flock of geese, with an occasional rattling or scraping sort of noise.

Two non-commissioned officers of the Royal Artillery were standing by me the whole time and would gladly have approached close to the string had not I convinced them of the danger of touching or even coming near the string during the storm. We anxiously watched what was going on at the other end of the string and the display was beautiful beyond description. The reel was occasionally enveloped in a blaze of purple arborized electric fire, whose numberless branches ramified over the silken cord and through the air to the blades of grass which also became luminous on their points and edges, over a surface of some yards in circumference. We also saw a complete globe of fire pass over the silken cord which appeared to be about the size of a musket ball and which was exceedingly brilliant. (*Phil. Mag.*, V.5, p. 418, 1834.)

The following account of electric kite experiments which were made by a local scientist in the north of England appeared in the Lancaster Gazette for April 5, 1834:

On March 29th, I had a favorable opportunity of demonstrating experimentally to several of my friends at Kirkby Lonsdale, who had attended my recent course of lectures at that place, that an abundance of the electric fluid usually attends hail and snow storms. During three hail storms I floated one of my silken electric kites, with a wired string about three hundred yards long, and insulated at its base in the usual way by means of a silken cord.

The kite was elevated in each case about ten minutes prior to the arrival of the storm and the state of the atmosphere was ascertained and was found to be so feeble that not the slightest spark could be observed. As, however, the cloud from which hail was falling approached the kite, the fluid from the string presented itself in brilliant sparks to the knuckle; and during the transit of the cloud, became so abundantly discharged to the wire that was presented to the string that it struck in rapid succession through a stratum of six inches of air, and through three inches of air it presented a splendid continuous stream of electric fire.

These appearances were exhibited in each experiment, but the display of the electric fire was the most magnificent in the second, which was during the fiercest hail storm of that day. During an early part of this storm the electric fluid made a continuous rattling noise down the kite string (in consequence of the wire being broken in several places), and darted from the reel at the inferior extremity, to greater distances than in the other experiments. In one instance it struck over a stick about a yard long, to the hand of a young man who was presenting it to the kite string. Although the remote end of the stick was in contact with the ground by means of a very wet string and consequently a considerable discharge must have passed down the wet string to earth, yet the shock was so violent as to make the young man reel and nearly fall. (*Phil. Mag.*, V. 5, p. 418, 1834.)

As noted above the first installations of lightning rods were made in 1752. One of the first to be made in America was on the tower of the state house which was later known as Independence Hall in Philadelphia. Franklin then endeavored to stimulate the introduction of his method of lightning protection by citing cases of serious lightning damage to structures which might have been avoided by the use of lightning rods. In 1749 Captain John Waddell reported to the Royal Society that his ship *Dover*, lately launched, had sailed from New York

for London when it encounted a very heavy storm in which electric discharges resembling great torches appeared at the mast heads followed by a severe lightning stroke. This shattered the main mast, disabled many of the crew and caused so much damage to the main body of the ship that it began to leak. They had great difficulty in reaching land, which was accentuated by the fact that the four compasses of the ship were demagnetized. (*Philosophical Transactions*, Vol. 46, p. 289.)

Franklin then sent a communication to the Royal Society showing how wire connections between the mast heads and the sea would have protected the ship and possibly would have entirely prevented the stroke. He also took that occasion to describe some interesting experiments that he had made in which he had magnetized steel needles by electrical means. (*Philosophical Transactions*, Vol. 47, p. 289.)

Many reports of severe damage from lightning were being discussed in the technical press of that day and Franklin contributed several of these. The following case was one of the most significant:

Being in the town of Newbery in New England I was shown the effect of lightning on their church which had been struck a few months before. The steeple was a square tower of wood reaching 75 feet up from the ground to the place where the bell hung, over which rose a taper spire, of wood likewise, reaching 75 feet higher to the vane or weather-cock. Near the bell was fixed an iron hammer to strike the hours; and from the tail of the hammer a wire went down through a small gimlet hole in the floor that the bell stood upon, and through a second floor in like manner; then horizontally under and near a plastered ceiling of that second floor, till it came near a plastered wall; then down by the side of that wall to a clock, which stood about 20 feet below the bell. The wire was not bigger than a common knitting needle.

The spire was split all to pieces by the lightning and the parts flung in all directions over the square in which the church stood, so that nothing remained above the bell. The lightning passed between the hammer and the clock on the above mentioned wire without hurting either of the floors or having any effect upon them except making the gimlet holes a little bigger, and without hurting the plastered wall, or any part of the building so far as the aforesaid wire and pendulum wire of the clock extended; which latter wire was about the size of a goose quill. From the end of the pendulum down quite to the ground the building was exceeding rent and damaged and some of the stones in the foundation wall torn out and thrown to the distance of 20 or 30 feet. No part of the aforementioned long small wire between the clock and the hammer could be found except about two inches that hung to the tail of the hammer and about as much that was fastened to the clock; the rest being exploded and its particles dissipated in smoke and air as gun powder is by common fire, and had only left a black smutty track on the plastering three or four inches broad, darkest in the middle and fainter towards the edges. All along the ceiling under which it passed and down the wall. These were effects and appearances on which I would only make a few remarks: viz:—

1. That lightning in its passage through a building will leave wood to pass as far as it can in metal, and not enter the wood again till the conductor of metal ceases. And the same I have observed in other instances as to walls of brick and stone.

2. The quantity of lightning that passed through this steeple must have been very great by its effects on the lofty spire above the bell and on the square tower all below the end of the clock pendulum.

3. Great as the quantity was, it was conducted by a small wire and clock pendulum without the least damage to the building so far as they extended.

4. The pendulum rod being of sufficient thickness conducted the lightning without damage to itself; but the small wire was totally distroyed.

5. Though the small wire was itself distroyed yet it had conducted the lightning with safety to the building.

6. From the whole it seems probable that if even a small wire had been extended from the spindle of the vane to earth, before the storm, no damage would have been done to the steeple by that stroke of lightning though the wire itself had been destroyed. (Letter from Franklin to M. Delibard in Paris, 1755, *Philosophical Transactions*, Vol. 49, p. 307.)

Franklin's activity in advising the adoption of his invention for the protection of property yielded rich results, and his lightning rods soon came into general use. In 1753 he received the Copley medal from the Royal Society of London for his scientific achievements and a little later was made an honorary member of the society by unanimous vote. At about the same time he also became a member of the Royal Academy of Science of Paris and within a few years was admitted to practically all of the learned societies of Europe. His Philadelphia friend, Ebenezer Kinnersley, whom it will be recalled was started by Franklin in his scientific career, wrote a long letter to Franklin in 1763, which was read before the Royal Society and in which he described many of his recent experiments, closing with this eulogy:

And now, Sir, I most heartily congratulate you on the pleasure you must have in finding your great and well grounded expectations so far fulfilled. May this method of security from the distructive violence of one of the most awful powers of nature meet with such success, as to induce every good and grateful heart to bless God for this important discovery. May the benefit thereof be diffused over the whole globe. May it extend to the latest posterity of mankind; and make the name of Franklin, like that of Newton immortal. (*Philosophical Transactions*, Vol. 53, p. 97.)

Franklin continued to correspond with his scientific friends regarding electrical matters, but from 1754, when he was appointed a delegate from Pennsylvania to the first General Assembly of all the colonies, up to the time of his death in 1790, he was unable to do any experimental work of his own, being completely absorbed in his distinguished career as statesman, diplomat and philanthropist.

THE CHARGE EFFECT IN RELATION TO THE KINETICS OF PHOTOGRAPHIC DEVELOPMENT. I. THE GENERAL EFFECT.

BY

T. H. JAMES.

Communication No. 1026 from the Kodak Research Laboratories.

The view is rather widely accepted [1–5] that the reduction of silver ions in normal "chemical" (direct) development takes place at or near the interface between silver halide and silver. At the start of development, the interface is that between silver halide and the latent-image material (very probably silver); in the later stages of development of the individual grains, the silver involved is that formed by prior reduction of part of the silver halide.

Once the interface is accepted as the locale of the reduction of silver ions, it remains to be decided whether or not the developing agent must be present at the interface before the reduction can occur. On the basis of what may be termed the catalytic hypothesis, development depends primarily upon the adsorption of silver ions to the silver and the accompanying alteration in the activation energy conditions of the reaction between the silver ions and the developing agent. An alternative view holds that the silver nuclei act simply as electrodes which can accept electrons from the developer at any point on the metal surface and can conduct the electrons to the interface. The pertinent experimental evidence does not allow a clear-cut decision between these views, but the trend is against the electrode hypothesis. The evidence on the adsorption of hydroquinone, in particular, seems incompatible with the electrode scheme. The kinetics of development by hydroquinone indicate that adsorption of the bivalent ion occurs prior to the reaction.[6] However, both direct adsorption measurements [7] and kinetic measurements [8] fail to indicate adsorption of hydroquinone to silver. The obvious implication is that the hydroquinone is adsorbed (as bivalent ion) to the silver bromide surface or to the interface. There is indirect evidence for the adsorption to silver bromide,[9] and it

[1] S. E. Sheppard, *Phot. J.*, 69, 330 (1929).
[2] A. J. Rabinovitch, *Trans. Faraday Soc.*, 34, 920 (1938).
[3] T. H. James and G. Kornfeld, *Chem. Rev.*, 30, 1 (1942).
[4] W. F. Berg, *Trans. Faraday Soc.*, 39, 115 (1943).
[5] Kh. S. Bagdasar'yan, *J. Phys. Chem.*, USSR, 17, 336, No. 5–6 (1943).
[6] T. H. James, *J. Phys. Chem.*, 44, 42 (1940).
[7] E. S. Perry, A. Ballard, and S. E. Sheppard, *J. Am. Chem. Soc.*, 63, 2357 (1941).
[8] T. H. James, *J. Am. Chem. Soc.*, 61, 648 (1939); *J. Phys. Chem.*, 45, 223 (1941).
[9] P. Wulff and K. Seidl, *Z. wiss. Phot.*, 28, 239 (1930).

seems probable that hydroquinone adsorbed substantially at the interface is responsible for the development reaction.

Regardless of whether or not the developing agent must actually contact the silver ions before reaction can occur, the developer ions or molecules must still reach the grain surface (silver halide or silver). Many of the developing agents are active only in the form of ions, and in such cases the approach of the ions to the surface is opposed by a negative potential in the vicinity. This "charge barrier" protecting the grain surface does not play a fundamental rôle in the mechanism of development, as was once supposed,[10] but it does modify the kinetics of development *by the ionic agents* in several important respects.

Two separate sources contribute to the electric field through which the developer ion must pass. The gelatin molecules themselves possess a net charge when the pH of the layer is other than that corresponding to the isoelectric point. In the presence of the conventional alkaline developing solution, the gelatin molecules are negatively charged. A second source of charge lies in the excess halide ions which are adsorbed to the grain surface in the normal photographic emulsion. This charge, although originating with the ions adsorbed to the silver halide, will extend in some degree over the latent-image nuclei or the silver nuclei formed by partial development of the grain. A localized drop in potential will occur over the silver nuclei, however, particularly the larger ones, and such a drop will behave as a weak point in the charge barrier protecting the surface from the approach of negatively charged ions.

The effectiveness of the charge barrier in protecting the grain surface from contact with the developer ions will depend both upon the magnitude of the electric potential and upon the charge of the developer ions. The greater the potential, the smaller will be the *percentage* of developer ions which have sufficient kinetic energy (Boltzmann distribution) to penetrate the barrier; likewise, the greater the charge of the developer ion, the smaller will be the percentage of those ions which can penetrate a barrier of given magnitude. Any change which reduces the magnitude of the barrier will increase the percentage of ions which can penetrate it, and consequently will increase the rate of development by an ionic developing agent, other things remaining constant. Conversely, any change which increases the magnitude of the barrier will decrease the rate of development.

The kinetic effects which originate in the action of the charge barrier fall into two somewhat overlapping groups. Those in the first group are produced primarily by a general action of the charge barrier or by a general change in the magnitude of the barrier. Those in the second group originate from a localized change in the barrier occurring in the vicinity of the growing silver specks. In the first group belong: the

[10] G. Schwarz and F. Urbach, *Z. wiss. Phot.*, 31, 77 (1932).

accelerating action which, under certain conditions, neutral salt exerts upon development; the accelerating action of certain dyes, such as phenosafranin and pinacyanol (the latter in amounts insufficient to cover the silver bromide grain surface completely); and the variation in the relative rates of development by agents of similar structure but different charge. In the second group belong the induction period in development and various effects derived from the induction period. Among the latter are the increase in emulsion speed (for incomplete development) obtained when sulfite is added to solutions of charged agents, the excessively high contrast obtained under certain circumstances when development is carried out in a caustic hydroquinone solution in the absence of sulfite, and the "special" properties of certain complex developers. The present paper will deal only with the kinetic effects belonging to the first group. The induction period and effects derived from it will be considered in subsequent papers.

The known developing agents fall into one of the following charge classes: The agents of one important group, comprising many derivatives of *p*-phenylenediamine, react in the neutral molecular state (*p*-phenylenediamine itself is not included, since, although it reacts in the neutral state, it acts primarily as a physical developer). The agents of a second group react in the form of the univalent ion. Included in this group are: *p*-aminophenol, methyl-*p*-aminophenol (known in the form of the sulfate as "elon" or "metol"), catechol, and *p*-aminophenylglycine. A third group, covering the doubly charged ions, includes hydroquinone and most of its derivatives, methyl-*p*-aminophenol monosulfonic acid, ascorbic acid, *p*-hydroxyphenylglycine (glycin), and the ferro-oxalate ion. The hydroquinone monosulfonate ion has three negative charges, and the disulfonate has four. (It is probable that ferrous ion is a developing agent,[11] and this ion, of course, has a double positive charge.)

SALT EFFECT.

The first (outer) barrier through which the developer ion must penetrate is the gelatin layer. The negative charge of the gelatin (on the alkaline side of the isoelectric point) markedly retards development by the negatively charged agents when the total salt concentration is low. Neutral salt depresses the charge of the gelatin, and consequently increases the rate of development. This salt effect, first noted by Lüppo-Cramer,[12] is concerned almost exclusively with the gelatin in the normal photographic emulsion. Only a very small portion of the increase which salt produces in the development rate can be attributed to a depression of the bromide barrier.[13]

[11] H. Ammann-Brass, *Phot. Ind.*, 35, 827 (1937).
[12] H. Lüppo-Cramer, *Kolloid-Z.*, 15, 164 (1914).
[13] T. H. James, *J. Chem. Physics*, 12, 453 (1944).

The salt effect increases with the charge of the developing agent. A good illustration is supplied by results obtained with the series of structurally similar agents: sym-dimethyl-*p*-phenylenediamine, methyl-*p*-aminophenol, methyl-*p*-aminophenol sulfonate. The ionic charges in this series are 0, −1, and −2, respectively. The data given in Table I

TABLE I.

Dependence of the Salt Effect upon the Charge of the Developing Agent.

	$1/t$	*R* Values for log *E* of		
		0.55	1.15	1.75
s-dimethyl-*p*-phenylenediamine				
(1) with no added salt	1.0	0.10	0.115	0.14
(2) with 0.667*M* KNO_3	0.91	.11	.13	.15
(3) with 0.667*M* Na_2SO_4	.91	.10	.12	.13
Ratio: 2/1	.91	1.10	1.10	1.07
3/1	.91	1.00	1.05	0.93
methyl-*p*-aminophenol				
(1) with no added salt	0.53	0.090	0.12	0.14
(2) with 0.667*M* KNO_3	1.05	.135	.175	.19
(3) with 0.667*M* Na_2SO_4	0.71	.115	.16	.185
Ratio: 2/1	2.0	1.50	1.46	1.36
3/1	1.43	1.28	1.33	1.32
methyl-*p*-aminophenol sulfonate				
(1) with no added salt	0.088	0.022	0.040	0.065
(2) with 0.667*M* KNO_3	.232	.052	.094	.124
(3) with 0.667*M* Na_2SO_4	.167	.039	.072	.102
Ratio: 2/1	2.65	2.36	2.35	1.90
3/1	1.90	1.76	1.80	1.54

were obtained with solutions containing 0.005*M* developing agent, 0.04*M* excess potassium hydroxide, and 0.00667*M* potassium bromide, but no sulfite or other additions. Motion-picture positive film (Emulsion "C") was used, and all operations were carried out under an oxygen-free atmosphere at 20° C. The rates designated as "*R*" are measured by the slope of the density-time of development curves at a fixed density (0.4 for exposures of $\log E = 1.15$ and 1.75; 0.3 for $\log E = 0.55$). The $1/t$ values are measured by the reciprocals of the times required to obtain a density of 0.2 at an exposure of $\log E = 1.15$.

The oxidation products of many developing agents affect the development rates, and thus complicate any attempt at a quantitative study of the salt effect with those agents. The oxidation products of hydroquinone and certain other doubly charged agents accelerate development and this effect is probably the consequence of an action of the oxidation product, or some decomposition product thereof, upon the bromide barrier.[14] Accordingly, it is desirable in a quantitative investigation to use only those agents for which the oxidation product is without effect upon the barrier.

[14] T. H. James, *J. Phys. Chem.*, 43, 701 (1939).

The ferro-oxalate developer can be used in slightly acid and slightly alkaline solutions, but complications arise from other sources if the pH exceeds about 7.5. Ascorbic and iso-ascorbic acids are well suited for use at high pH values [15] and under conditions more nearly comparable to those existing with the conventional organic developers. In the initial investigation of development by these agents,[15] it was shown that the salt effect increased with increasing salt concentration. A graphical extrapolation of the data to zero salt concentration indicated that the development rate in the absence of salt would be extremely low. However, the data were not sufficient to show whether such an extrapolation was justified.

In the present investigation, the previous results have been extended. Figure 1 represents some of the data obtained for the effect of various

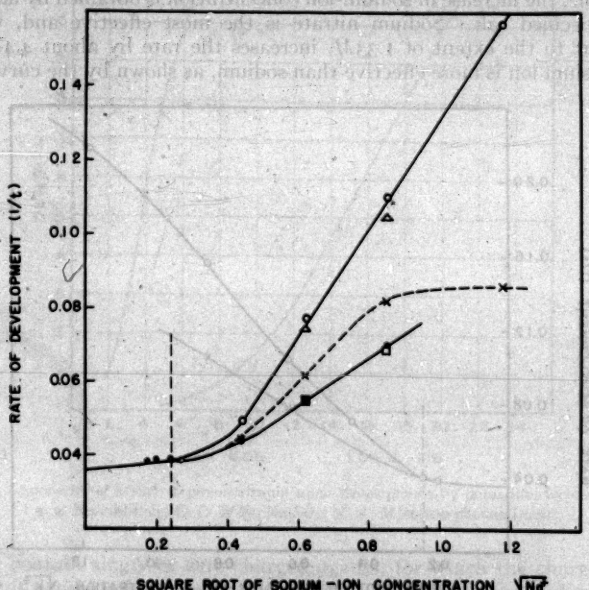

FIG. 1. Effect of sodium salts upon the rate of development by iso-ascorbic acid. ● ● OH^-; ○ ○ NO_3^-; × × $SO_4^=$; △ △ $C_2H_3O_2^-$; □ □ $CO_3^=$.

sodium salts upon development by 0.005M iso-ascorbic acid in caustic solution and in the presence of 0.0067M potassium bromide. The lower solid curve shows the variation in rate with changing concentra-

[15] T. H. James, *J. Am. Chem. Soc.*, 66, 91 (1944).

tion of sodium hydroxide. Development rate is plotted against the square root of the sodium-ion concentration. The rate shows little variation with the concentration below 0.05*M*. Extrapolation of these data to zero sodium-ion concentration suggests that the rate in the absence of salt is still substantial and, in this case, is equal to about 0.035. Above 0.05*M* sodium hydroxide, the rate increases with increasing concentration of hydroxide. This is a genuine salt effect, and not a pH effect, since the ionization of the iso-ascorbic acid is largely complete even at 0.05*M* hydroxide.

The other curves in Figure 1 show the effect of adding various sodium salts to a developer containing 0.05*M* sodium hydroxide. The sodium-ion concentration corresponding to this basic developer is indicated by the vertical broken line. For all points to the right of this line, the increase in sodium-ion concentration is obtained by adding the specified salt. Sodium nitrate is the most effective and, when present to the extent of 1.33*M*, increases the rate by about 4.4-fold. Potassium ion is more effective than sodium, as shown by the curves in

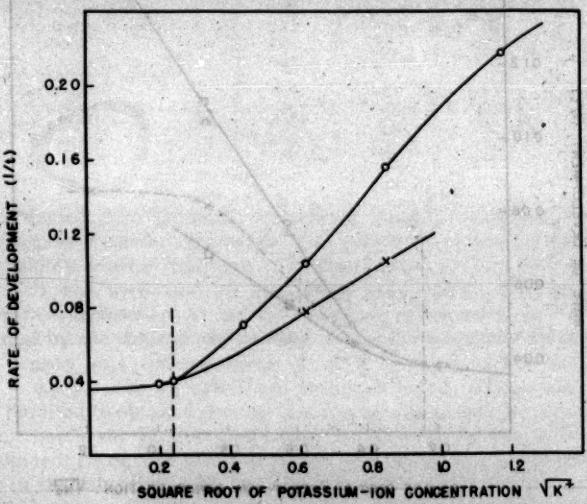

FIG. 2. Effect of potassium salts upon the rate of development by iso-ascorbic acid. ○ ○ NO_3^-; × × $SO_4^=$.

Figure 2. A concentration of 1.33*M* potassium nitrate increases the development rate by 6.2-fold.

The preceding data for the salt effect were all obtained at a pH of over 12. The magnitude of the effect, however, should be about the

same over the pH range from 8 to 12, since the charge of the gelatin layer increases only slightly with pH over this range. Values previously reported for iso-ascorbic acid at pH 10.3 show as large a salt effect as the present value for 12.7.

In most conventional developer formulas, moderate changes in salt concentration will not exert much influence on the development rate from a charge effect alone. The normal salt concentration is already high, being of the order of 1*M*. Moreover, most conventional devel-

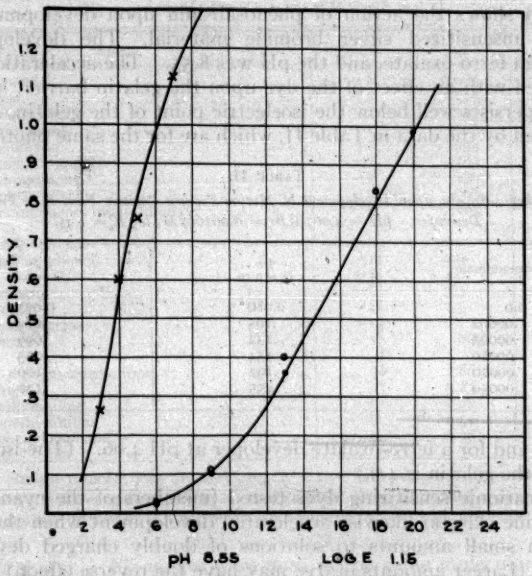

FIG. 3. Accelerating action of phenosafranin upon development by potassium ferro-oxalate. ● ● No addition; ○ ○ *M*/80 Na_2SO_3; × × *M*/20,000 phenosafranin.

opers contain singly or zero-charged agents, for which the charge salt effect is considerably smaller than it is for the doubly charged agents. When a doubly charged agent such as hydroquinone is used, it is usually employed in conjunction with a singly charged agent such as elon. In practice, factors other than the charge effect are sometimes operative. If the developing agent is only partially ionized, changes in salt concentration may change the degree of ionization and the pH of the solution. In the case of rapid developers, where diffusion plays a prominent rôle in determining the over-all rate, large amounts of salt

may depress the swelling of the gelatin sufficiently to bring about a substantial *decrease* in the rate of development.

SUBSTANCES PRODUCING A GENERAL REDUCTION IN THE BROMIDE BARRIER.

Development by charged agents, particularly those having at least two negative charges, may be accelerated by the addition of certain basic or cationic dyes which decrease the bromide barrier by becoming adsorbed themselves to the silver bromide surface. Typical dyes are the desensitizer, phenosafranin, and the optical sensitizer, pinacyanol. Figure 3 shows the action of phenosafranin upon development of a simple, unsensitized silver bromide material. The developer was potassium ferro-oxalate, and the pH was 8.55. The acceleration is not connected with an effect of the dye upon the gelatin barrier, however, since it persists well below the isoelectric point of the gelatin. This is illustrated by the data in Table II, which are for the same photographic

TABLE II.

Effect of Phenosafranin upon Development of Motion-Picture Positive Film in a Ferro-Oxalate Developer. pH = 4.06; KBr = 0.001667 M; log E'_4 = 1.45.

Phenosafranin	$1/t$ $D = 0.20$	R $D = 0.80$
0.0	0.080	0.075
.00002	.105	.084
.00005	.132	.099
.00010	.154	.095
.00020	.192	.100
.00050	.455	.126

material and for a ferro-oxalate developer at pH 4.06. (The isoelectric point of the gelatin is 4.9.)

The cationic sensitizing dyes tested (members of the cyanine and thiacyanine classes) likewise accelerated development when they were added in small amounts to solutions of doubly charged developing agents. (Larger amounts of dye may have the reverse effect.) These dyes, which are strongly adsorbed by silver bromide in the presence of excess bromide, undoubtedly also decrease the magnitude of the bromide barrier. It is probably for this reason that charge effects depending upon the bromide barrier are less pronounced when optically sensitized emulsions are used than when corresponding unsensitized emulsions are used. The barrier surrounding the grains in the sensitized emulsion is already depressed by the adsorbed sensitizing dye, and a further decrease in the barrier is relatively less important.

ABSOLUTE RATE OF DEVELOPMENT.

As long as any barrier (either bromide or gelatin) remains, it will act to decrease the rate of development by a charged agent. This action

will be considerably greater, the higher the charge of the developing agent. Of course, the charge is only one factor governing the rate of development. Individual differences in adsorption and in activation energies of reaction are to be expected. Even the oxidation products may introduce complications in some cases. However, a comparison of the rate data given in Table III shows a genuine correlation between

TABLE III.

Rate of Development of Motion-Picture Positive Film. Developing Agent = 0.005 M; pH = 12.7; KBr = 0.0067 M; Na_2SO_3 = 0.125 M; log E = 1.15.

Agent	Charge	$1/t$	R
iso-ascorbic acid	−2	0.061	0.083
hydroquinone	−2	.125	.184
chlorohydroquinone	−2	.125	.184
bromohydroquinone	−2	.175	.180
toluohydroquinone	−2	.294	.264
3-chlorotoluohydroquinone	−2	.262	.180
dichlorohydroquinone	−2	.139	.168
p-hydroxyphenylglycine	−2	.063	.200
methyl-*p*-aminophenol monosulfonate	−2	.667	.300
methyl-*p*-aminophenol	−1	4.0	.48
p-dimethylaminophenol	−1	2.9	.41
catechol *	−1	4.0	.45
p-aminocarvacrol	−1	3.3	.40
p-aminophenol	−1	2.5	.38
p-aminophenylglycine	−1	0.10	.084
hydroxylamine	−1	.061	.041
s-dimethyl-*p*-phenylenediamine	0	2.86	.39
diaminodurene	0	3.23	.45
p-aminodiethylaniline	0	2.22	.40
p-aminodimethylaniline	0	0.91	.26
methyl-*p*-phenylenediamine	0	3.33	.56

* The *bivalent* catechol ions are undoubtedly active in development, but the univalent ions are also quite active and are present in considerable concentration, thus giving the catechol solution many characteristics of a univalent developer even at this high pH value.

rate and charge. All agents were used in the same molecular concentrations, and at pH = 12.7. This pH value is sufficiently high to insure that the major portion of each agent is present in its active form (i.e., as the ion for each charged agent, or the free base for each agent which acts in the non-ionized form).

The differences between the $1/t$ rates of development by the doubly charged and singly charged agents, as shown in Table III, are quite marked. The differences in the R-values are considerably smaller, as is to be expected from considerations of the origin of the induction period (see subsequent paper). Hydroxylamine and *p*-aminophenylglycine, among the singly charged agents, give exceptionally small $1/t$ values for that group. These, however, are the agents which were observed previously to give abnormally low maximum emulsion speed

values.[16] These agents evidently require larger latent-image centers than the others, and this could account for a lower initial development rate, since fewer centers and fewer grains would be involved. Of much greater fundamental importance than the behavior of these exceptional agents is the fact that *no doubly charged agent has been found which, in the early stages of development, approaches the rates of development attained by the majority of the singly charged and uncharged agents.*

The uncharged agents do not show significantly greater rates than the singly charged agents, except in the one case where the singly charged agent is a member of the *p*-phenylenediamine series. A marked drop in reaction rate occurs when a negative charge is added by replacing the methyl or ethyl group in a substituted *p*-phenylenediamine by the acetic acid group.

The halogen derivatives of hydroquinone show little difference in rate from the parent substance under the present conditions, although a considerable variation had been noted previously [6] when development was carried out at pH = 8.9. It seems probable that the difference at the low pH value is due primarily to a difference in the concentration of the effective bivalent ion. The pH of 8.9 is well below the pK value for the second dissociation constant of all of the hydroquinones, and is below the pK value for the first dissociation constant of some of them. On the other hand, the present experimental conditions permit a comparison of the rates of development at approximately equal ionic concentrations. This could not be made at the lower pH value, since the dissociation constants of several of the hydroquinones are not known with sufficient accuracy.

ROCHESTER, N. Y.,
March 27, 1945.

[16] T. H. James, *J. Frank. Inst.*, **239**, 41 (1945).

ON PARAMETRIC EXCITATION.

BY

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1. INTRODUCTION.

The possibility of exciting a dynamical system (electrical or mechanical) by means of a periodic variation of its parameters such as capacity, inductance, spring constant and so on, has been known for a long time. Lord Rayleigh (1), for example, analyzes Melde's experiment (2) in which one end of a stretched string is attached to a prong of a tuning fork vibrating in the direction of the string. In this experiment the oscillations of the tuning fork produce periodic fluctuations in the tension of the string which, in turn, cause transverse vibrations of the string.

M. Brillouin (3) and H. Poincaré (4) investigated similar effects in electric circuits. More recently these phenomena have been studied by a group of Russian scientists under the leadership of L. Mandelstam and N. Papalexi (5) who correlated these phenomena with the theory of subharmonic resonance. In view of the fact that these phenomena of self-excitation are generally associated with the periodic variation of certain parameters of the system, the term *parametric excitation* is commonly used in the current literature on this subject. The investigations of Mandelstam and Papalexi resulted in the development of an interesting electric machine, the parametric generator (6), capable of delivering a considerable power output. This machine transforms the mechanical power absorbed in the process of a periodic variation of its parameter into electrical energy of an alternating current of a certain frequency. The mathematical theory of these phenomena is closely related to the theory of the differential equations with periodic coefficients, more specifically, to the equations of the Mathieu-Hill type (7).

Most of the literature on this subject deals, however, with the periodic solutions of these equations and very little is known about the unstable solutions which are of considerable interest from the point of view of parametric excitation. Moreover, only *linear* differential equations with periodic coefficients have been investigated so far. In applications one always comes across *non-linear* equations of this kind. In fact, the unstable solutions of the linear equations increase indefinitely and can not, therefore, characterize a stationary periodic process.

Mandelstam and Papalexi obtained an interesting confirmation of this fact in their experiments. They report that if the circuit of the parametric generator is devoid of non-linear elements, the voltage

builds up to a high value at which the insulation is punctured. By providing a non-linear element, on the contrary, it is possible to obtain a stable performance of the generator. Unfortunately there exists no theory of non-linear differential equations with periodic coefficients and the most one can do is to study the conditions of stability of the solutions in the linear region. Important contributions to these studies were made by E. Meissner (8) and by B. Van der Pol and M. S. O. Strutt (9). These authors have explored the zones of stable and unstable solutions of the Mathieu-Hill equation although the actual form of the unstable solutions and the effect of the relative frequency of the parameter variation have received very little attention.

It will be shown in what follows that by transferring the problem to the phase plane, it is possible to extend the qualitative study of solutions of these equations so as to be able to approach the problem from a more general standpoint.

2. EQUATIONS OF MATHIEU AND HILL.

The differential equation of Hill is

$$\ddot{x} + F(t)x = 0, \tag{1}$$

where $F(t)$ is a periodic function with period 2π. The Mathieu equation is

$$\ddot{x} + (a^2 + b^2 \cos t)x = 0. \tag{2}$$

A particular case of the Hill equation is the so called Hill-Meissner equation in which the function $F(t)$ is a rectangular "ripple" shown in Fig. 1. The Fourier expansion in this case is

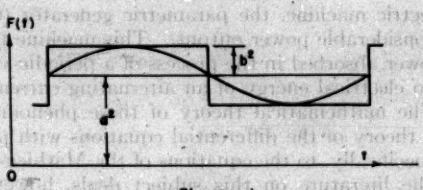

Figure 1

$$F(t) = a^2 + \frac{4}{\pi} b_0^2 \left(\cos t - \frac{1}{3} \cos 3t + \frac{1}{5} \cos 5t - \cdots \right). \tag{3}$$

The theory of Hill's equation is very similar to that of the Mathieu equation so that both equations, to some extent, are studied together. Although Hill's equation appears to be more complicated than Mathieu's equation owing to the presence of the Fourier series, in some cases the discussion is facilitated by the particular form (3) of the function $F(t)$ as was pointed out by Meissner, Van der Pol and Strutt (loc. cit.).

We shall make an extensive use of this remark. The zones of stability of solutions of the Mathieu equation and of the Hill-Meissner equation are of the same general character as was pointed out by Strutt (10) and it seems likely that on the average the unstable solutions of both equations are not much apart from each other. The experimental evidence seems to corroborate this conclusion although no direct quantitative comparison of the unstable solutions of equations of both types has been attempted so far.

We shall first analyse the quantitative aspects of the integral curves, the *phase trajectories* of the Hill-Meissner equations, by transferring the problem to the phase plane of the variables $(x, \dot{x})$ and will return to the Mathieu equation in a later section.

It is convenient to write the Hill-Meissner equation in the form

$$\ddot{x} + (a^2 \pm b^2)x = 0, \tag{4}$$

which means that we consider alternatively the two equations

$$\ddot{x} + (a^2 + b^2)x = 0; \qquad \ddot{x} + (a^2 - b^2)x = 0, \tag{5}$$

during each half period π of the ripple with the understanding that solutions have to be continuous on physical grounds although not necessarily analytic at the points at which the changes from $(a^2 + b^2)$ to $(a^2 - b^2)$ occur. We will assume that $a^2 > b^2$ inasmuch as we will be concerned with the problem of *modulation* of the quantity a^2 by a rectangular ripple b^2.

3. PARAMETRIC EXCITATION OF A NON-DISSIPATIVE CIRCUIT.

It is useful to approach the problem of parametric excitation in the simple case of a non-dissipative circuit whose differential equation is

$$L_0 \frac{d^2q}{dt^2} + \frac{1}{C_0} q = 0, \tag{6}$$

where L_0 and C_0 are the inductance and the capacity respectively and q is the quantity of electricity stored in the condenser. If a certain charge q_0 exists initially and the circuit is closed an undamped oscillation of frequency $\omega_0 = 1/\sqrt{L_0C_0}$ will be established. The energy will oscillate indefinitely between the potential form $C_0V_0^2/2$, V_0 being the potential difference across the condenser when the current $i = dq/dt$ is zero, and the electromagnetic form $L_0i_0^2/2$ when the condenser is discharged and the current is a maximum. The energy stored in the system is $E_0 = (1/2C_0)q_0^2$. The maximum values q_0 and i_0 are given by the expressions $q_0 = C_0V_0$ and $i_0 = q_0\omega_0$.

We shall now assume that the capacity C_0 is "modulated" by a rectangular ripple $\pm \Delta C$ between the limits $C_{max.} = C_0 + \Delta C$ and $C_{min.} = C_0 - \Delta C$, $(\Delta C \ll C_0)$. Let us start from the instant when the whole energy is electrostatic and reduce the capacity $(-\Delta C)$ suddenly.

To accomplish this, impulsive mechanical work will be required to overcome the electrostatic forces. Since the system is conservative by our assumption, an equivalent increment, ΔE_1, of electric energy will be added. Assuming ΔC small, this increment is obtained by differentiating the expression $E = (1/2C)q_0^2$ with respect to C which gives $\Delta E_1 = -(q_0^2/2C_0^2)\Delta C$. The increment is positive if $\Delta C < 0$, that is, if the capacity is decreased. After this operation the energy stored in the condenser will be

$$E_1 = E_0 + \Delta E_1 = \frac{q_0^2}{2C_0}\left(1 + \frac{|\Delta C|}{C_0}\right) = \frac{q_0^2}{2C_0}a, \qquad (7)$$

where $a = 1 + (|\Delta C|/C_0) > 1$. A quarter period later when the energy is purely electromagnetic ($q_0 = 0$; $i = dq/dt$ maximum) we can reestablish the former value of the capacity by increasing it by $+\Delta C$ without doing any work. At $t = T/2$, when the energy is again purely electrostatic, the capacity is again reduced ($-\Delta C$) which adds another increment of energy $\Delta E_2 = q_0^2 a/2C_0^2$, so that the new energy content is now

$$E_2 = E_1 + \Delta E_2 = \frac{q_0^2 a}{2C_0}\left(1 + \frac{|\Delta C|}{C_0}\right) = \frac{q_0^2}{2C_0}a^2 \qquad (8)$$

and so on. It is seen that by timing the discontinuous changes of capacity in the manner shown the energy stored in the system will gradually increase as the result of the operation of the ripple $\pm \Delta C$. One verifies easily, that if instead of *decreasing* the capacity ($-\Delta C$) at the instants 0, $T/2$, T, $\cdots$ the capacity is *increased* ($+\Delta C$) at these instants, the operation of the ripple $\pm \Delta C$ will *withdraw* the energy from the system.

In practice instead of discontinuous changes $\pm \Delta C$ of capacity a continuous modulation $C = C_0 + \Delta C \sin 2\omega t$, ($\omega = 2\pi/T$) is employed. The results obtained are found to be in qualitative agreement with the above conclusions obtained on the basis of a discontinuous variation of capacity. This shows that on the average the solutions of the Mathieu equations are not far apart from those of the Hill-Meissner equation. The latter is simpler to discuss, as will appear later, so we will proceed with the discontinuous method. It is to be noted also that in both cases (continuous and discontinuous) the frequency of modulation is twice that of the oscillatory process in the circuit. We shall later consider a more general situation.

The argument remains the same if instead of the capacity variations ($\pm \Delta C$) the inductance variations $\pm \Delta L$ are used. The timing of the ripple in this case is exactly the same as before, viz: the coefficient of the inductance is decreased ($-\Delta L$) at the instants 0, $T/2$, $\cdots$ and increased ($+\Delta L$) at the instants $T/4$, $3T/4$, $\cdots$. To the same timing there will correspond, however, a diametrically opposite effect, viz. at

the instants $T/4$, $3T/4$, when L is increased there will be an addition of energy since the whole energy is electromagnetic at these instants, whereas at 0, $T/2$, $\cdots$ when L is decreased no work will be done since the electromagnetic energy is zero.

4. TOPOLOGY OF THE HILL-MEISSNER EQUATION.

We shall write equation (6) taking into account the step-wise modulation of capacity as

$$\ddot{q} + \frac{1}{L_0 C_0 (1 \pm \gamma_c)} q = 0, \tag{9}$$

where $\gamma_c = \Delta C/C_0$ is the index of modulation. Putting $1/L_0C_0 = \omega_0^2$ and considering $\gamma_c \ll 1$ without any loss of generality equation (9) becomes

$$\ddot{q} + \omega_0^2(1 \mp \gamma_c)q = 0.$$

Introducing a new independent variable $\tau = \omega_0 t$ we have

$$\frac{d^2q}{d\tau^2} + (1 \mp \gamma_c)q = 0. \tag{10}$$

This equation should be considered as an alternate sequence of the two equations

$$\frac{d^2q}{d\tau^2} + \alpha_1^2 q = 0 \quad \text{and} \quad \frac{d^2q}{d\tau^2} + \alpha_2^2 q = 0, \tag{11}$$

where $\alpha_2^2 = 1 + \gamma_c$; $\alpha_2^2 = 1 - \gamma_c$, replacing each other at the frequency of the ripple $\pm \Delta C$. Since the solutions $q(\tau)$ of these equations have to be continuous, the problem reduces to fitting the solutions of both equations by continuity at the end of the intervals $(+\Delta C, -\Delta C, +\Delta C \cdots)$. The solution $q(\tau)$ as a whole is thus a continuous function of time although not analytic at the points where C varies discontinuously. In some cases this loss of analyticity appears as a discontinuity in the second derivative, the first derivative being continuous; in other cases the discontinuity may occur in the first derivative of the solution $q(\tau)$.

We will now transfer the problem to the phase plane of the variables q, $dq/d\tau$. The solutions $q(\tau)$ will then be represented by the integral curves, or *phase trajectories* of equations (11), and the dynamical process described by these equations will be represented by the *motion* of the *representative point* R on the trajectories as shown in Fig. 2.

For $\gamma_c = 0$, $\alpha_1^2 = \alpha_2^2 = 1$ the trajectories of equations (11) form a continuous family Γ_0 of concentric circles with the origin O as center. If $\gamma_c \neq 0$, the trajectories form continuous families Γ_1 and Γ_2 of concentric homotethic ellipses shown in Fig. 2. The family Γ_1 corresponding to $\alpha_1^2 > 1$ has a constant ratio $b/a = \alpha_1^2 = 1 + \gamma_c$ of semiaxes

and for the family Γ_2 this ratio is $b'/a = \alpha_2^2 = 1 - \gamma_c$. The family Γ_1 corresponds to the reduced value $C_0 - \Delta C$ of capacity and Γ_2 to the increased value $C_0 + \Delta C$. The origin O is the singular point of the differential equations (11). The two families Γ_1 and Γ_2 thus serve as a kind of reference systems determining the motion in the phase plane. For instance, if for $t = 0$ certain initial conditions, say q_0, O are given and the value of C is prescribed, e.g. $C = C_0 - \Delta C$, the process is depicted by the motion of starting from the point A corresponding to the initial conditions and moving along the ellipse of the family Γ_1, passing through that point. If a later instant $t = t_1$, to which corresponds the point B on the ellipse, the capacity is changed and is $C = C_0 + \Delta C$, the representative point will pass on the elliptic trajectory belonging to the family Γ_2 passing through B and will continue

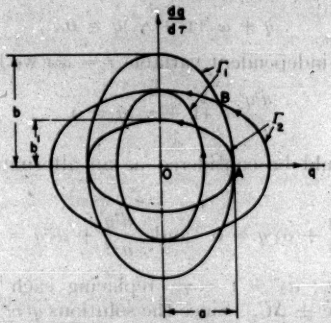

Figure 2

to move on that trajectory until the next change ($C = C_0 - \Delta C$) and so on.

We shall see in the following sections that this geometrical representation of the solutions $q(\tau)$ of equation (9) is very convenient and permits obtaining an account of the various features of the dynamical process. It is apparent that whenever a piece-wise analytic trajectory formed by the elliptic arcs is closed and the path is reentrant * we encounter a periodic phenomenon.

As an example of the application of this method, we shall consider the case of the parametric excitation discussed in Section 3 in connection with the capacity ripple. Let us start from a point $A(q_0, O)$ after the capacity has been reduced ($C = C_0 - \Delta C$). The representative point,

* It will be shown in Section 6 that under certain conditions the path may be closed after one revolution (2π) of the radius vector without the reentrance of the path. In such cases the phenomenon is not periodic with period 2π but it may be periodic for 4π, 6π, ...

Fig. 3, will move on the arc AB of the elliptic trajectory of the family Γ_1. At the point B ($q = 0$, $dq/d\tau$ max.) the capacity is increased ($C = C_0 + \Delta C$) and the arc BC of the family Γ_2 is followed. At the point C the capacity is reduced and the next arc CD is of the family Γ_1, and so on. After one period 2π one reaches the point E corresponding to $q_{2\pi} > q_0$ which shows that the energy content of the system has been increased.

If at the point A the capacity is increased ($+ \Delta C$), at B' decreased, at C' increased and so on, a convergent piece-wise analytic spiral $AB'C' \cdots$ would be followed, in which case the ripple $\pm \Delta C$ *withdraws* the energy from the system.

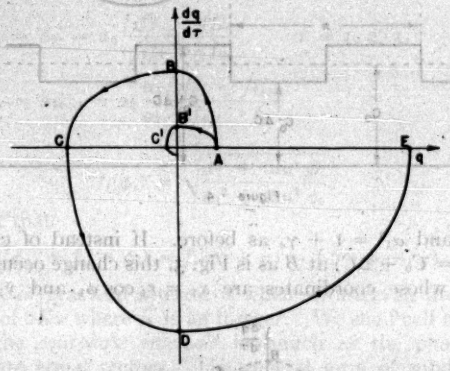

Figure 3

Similar conclusions can be obtained when the inductance L undergoes a step-wise modulation $L_0 \pm \Delta L$ taking into the account the observation made at the end of Section 3. We note that the loss of analyticity at the points B, C, D, $\cdots$ is due to the fact that the second derivatives arc discontinuous, although the first derivatives are continuous. In the general case the discontinuities occur in the first derivatives (e.g. point B in Fig. 2).

5. RIPPLES OF DIFFERENT FREQUENCIES AND PHASE ANGLES.

The preceding study was limited to a very special case when the "frequency" of the ripple is twice that of the circuit and when the changes $\pm \Delta C$ (or $\pm \Delta L$) occur at the instants 0, $T/4$, $T/2$, $3T/4 \cdots$ of the oscillation as shown in Fig. 4. We shall now investigate a more general situation. For this purpose, it is necessary to establish certain general relations governing the motion of the representative point on elliptic arcs of the two families Γ_1 and Γ_2. Suppose we start at the

point A Fig. 5, on elliptic trajectory E_1 (family Γ_1). The radius vector r is given by the equation

$$r = \frac{a_0}{\sqrt{\cos^2\phi + \frac{1}{\alpha_1^2}\sin^2\phi}}, \qquad (12)$$

where ϕ is the angle of the radius vector, $a_0 = OA$ the semi-axis (on

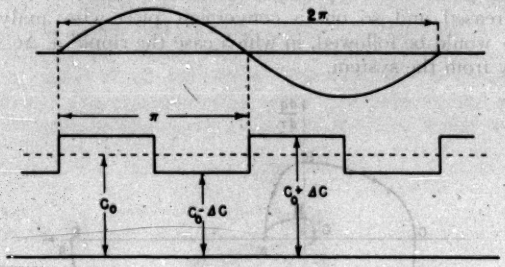

Figure 4

the x axis) and $\alpha_1^2 = 1 + \gamma_c$ as before. If instead of changing the capacity ($C = C_0 + \Delta C$) at B as is Fig. 3, this change occurs at a point M (Fig. 5) whose coordinates are $x_1 = r_1 \cos\phi_1$ and $y_1 = r_1 \sin\phi_1$,

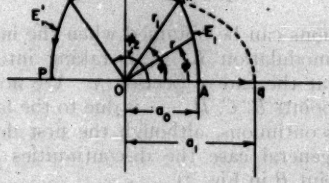

Figure 5

where r_1 is given by equation (12) in which $\phi = \phi_1$, the representative point will describe an elliptic arc MN of the family Γ_2. The x − semi-axis a_1 of the ellipse E_2 is obtained from the equation

$$\frac{x_1^2}{a_1^2} + \frac{y_1^2}{a_1^2\alpha_2^2} = 1$$

which expresses that the ellipse E_2 passes through M and has the ratio

of semi-axes $\alpha_2^2 = 1 - \gamma_c$. This gives

$$a_1 = \sqrt{x_1^2 + \frac{y_1^2}{\alpha_2^2}}, \tag{13}$$

where x_1^2 and y_1^2 has been previously calculated. At the point N of coordinates x_2, y_2 another elliptic trajectory E_1^1 (family Γ_1) starts and continues to a point P with coordinates x_3, y_3 and with parameter $\alpha_1^2 = 1 + \gamma_c$ and so on. Carrying out this procedure (we omit the calculations) one obtains the following general relations expressing a_N, the x-semiaxis of the ellipse after N changes of capacity, depending upon whether N is even or odd.

$$a_N = a_{2\nu} = a_0 \frac{f_1 f_3 \cdots f_{2\nu-1}}{f_2 f_4 \cdots f_{2\nu}}, \qquad \nu = 1, 2, 3, \cdots \tag{14}$$

$$a_N = a_{2\nu+1} = a_0 \frac{\alpha_1 f_1 f_3 \cdots f_{2\nu+1}}{\alpha_2 f_2 f_4 \cdots f_{2\nu}}, \tag{15}$$

where

$$f_i(\phi_i) = \sqrt{\frac{\alpha_2^2 + \tan^2 \phi_i}{\alpha_1^2 + \tan^2 \phi_i}}. \tag{16}$$

It is apparent that

$$f_i(\phi_i) = f_i(-\phi_i) = f_i(\phi_i + \pi) = f_i(-\phi_i + \pi). \tag{17}$$

The only case of practical interest is when all intervals are equal and are fractions of $2K\pi$ where K is an integer. We shall call this mode of subdivision the *equiphase intervals* inasmuch as the phase plane is subdivided into equal sectors. The general form of subdivisions for $N = 4$ for instance will be ϕ_0, $\phi_1 = \phi_0 + (\pi/2)$, $\phi_2 = \phi_0 + (2\pi/2)$, $\phi_3 = \phi_0 + (3\pi/2)$, $\phi_4 = \phi_0 + (4\pi/2) = \phi_0$, as is obvious from Fig. 6.

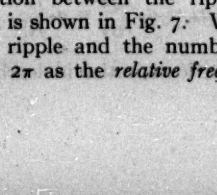

Figure 6

The actual phase relation between the ripple and the undamped oscillation in the circuit is shown in Fig. 7. We may call the angle ϕ_0 the *phase angle* of the ripple and the number N of changes $\pm \Delta C$ contained in the period 2π as the *relative frequency* of the ripple. A

complication arises, however, due to the fact that the equi-phase intervals as above defined are not the intervals of equal time, the *equi-time intervals*, owing to the non-uniformity of motion of the representative point on the elliptic trajectories. We shall come back to this question in Section 7. It is sufficient to mention here that the conclusions derived from the consideration of equi-phase intervals permit obtaining the principal features of phase trajectories of the Hill-Meissner equation in a very simple manner and the introduction of the equi-time intervals, while complicating the calculations, does not reveal any additional features of interest, except in some special cases.

6. EQUI-PHASE INTERVALS.

We shall examine now the various cases of parametric excitation in terms of the two parameters N and ϕ_0. It is convenient to consider separately the following groups of numbers (1) $N = 4\nu$;

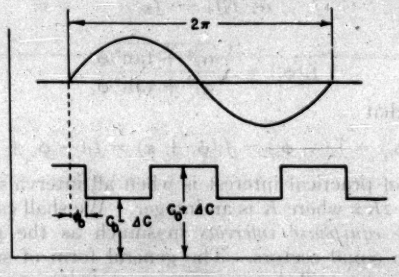

Figure 7

(2) $N = (2\nu + 1)2$; (3) $N = 2\nu + 1$; (4) $N = p/q$; where $\nu = 1, 2, 3, \cdots$ and p and q are relatively prime. In the first three groups N is an integer and in the last it is a rational fraction. This covers all cases of interest in practice.

(1) *First group* $N = 4, 8, 12 \cdots$.

Let us consider the first case $N = 4$ which has been studied in Section 3 by an elementary method; we shall now apply the general method of Section 5. The intervals are ϕ_0, $\phi_1 = \phi_0 + (\pi/2)$, $\phi_2 = \phi_0 + (2\pi/2)$, $\phi_3 = \phi_0 + (3\pi/2)$, $\phi_4 = \phi_0$, shown in Fig. 6. Hence by the properties (17) of the functions f we have $f_0 = f_2 = f_4$; $f_1 = f_3$. Using equation (14) we get

$$a_4 = a_0 \frac{f_1 f_3}{f_2 f_4} = a_0 \frac{f_1^2}{f_0^2}. \tag{18}$$

The condition for parametric excitation is clearly $f_1^2/f_0^2 \geqslant 1$. Replacing

f_1 and f_0 by their values (16) this condition is

$$\frac{\alpha_2^2 \tan^2 \phi_0 + 1}{\alpha_1^2 \tan^2 \phi_0 + 1} \geqslant \frac{\alpha_2^2 + \tan^2 \phi_0}{\alpha_1^2 + \tan^2 \phi_0}. \tag{19}$$

The equal sign corresponds to the closed trajectories. The condition for the closed trajectories is given by the equation $\tan^4 \phi_0 = 1$, that is for $\phi_0 = \pi/4,\ 3\pi/4,\ 5\pi/4$ and $7\pi/4$. The lines AC and BD in Fig. 8

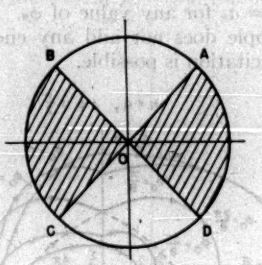

Figure 8

correspond to the closed trajectories, hence they are the thresholds at which the parametric excitation appears (or disappears). In order to determine the zones of ϕ_0 within which it exists, let us put $\phi_0 = 0$ (or $\phi_0 = \pi$) and compare both terms of the inequality (19). The left hand term is one and the right hand term is α_2^2/α_1^2 so that in these zones, shown in shading, the parametric excitation takes place. In the non-shaded sectors no parametric excitation occurs; one verifies this by putting $\phi_0 = \pi/2$ (or $\phi_0 = 3\pi/2$) in the inequality (19) which gives the left hand term α_2^2/α_1^2 and right hand term is unity which means that $a_4 < a_0$ so that no parametric excitation exists in this case. The "pumping" of the energy into the system by the ripple is maximum at $\phi_0 = 0$ or $\phi_0 = \pi$. Substituting these values into equation (18) one finds

$$a_4 = \frac{\alpha_1^2}{\alpha_2^2} a_0 = \frac{1 + \gamma}{1 - \gamma} a_0. \tag{20}$$

Defining δ as average decrement, clearly $a_4 = a_0 e^{2\pi\delta}$ whence

$$\delta = \frac{1}{2\pi} \log \frac{\alpha_1^2}{\alpha_2^2} = \frac{1}{2\pi} \log \left(\frac{1 + \gamma}{1 - \gamma} \right). \tag{21}$$

One can construct the trajectories for different values of ϕ_0 following the graphical procedure outlined in connection with Fig. 2. A few such trajectories are shown in Fig. 9 for $N = 4$ for several values of ϕ_0. For $N = 8, 12, 16, \cdots$ the conclusions remain the same but the number

of alternate zones in which the trajectories diverge (or converge) is accordingly greater.

(2) *Second group* $N = 6, 10, 14, 18, \cdots$. As an example, consider the case $\nu = 1$, $N = 6$

$$a_6 = a_0 \frac{f_1 f_3 f_5}{f_2 f_4 f_6}, \tag{22}$$

the intervals are $\phi_0, \phi_1 = \phi_0 + 2\pi/6, \cdots$ so that $f_0 = f_3, f_1 = f_4, f_2 = f_5$, which shows that $a_6 = a_0$ for any value of ϕ_0. This means that over the period 2π the ripple does not add any energy to the system so that no parametric excitation is possible.

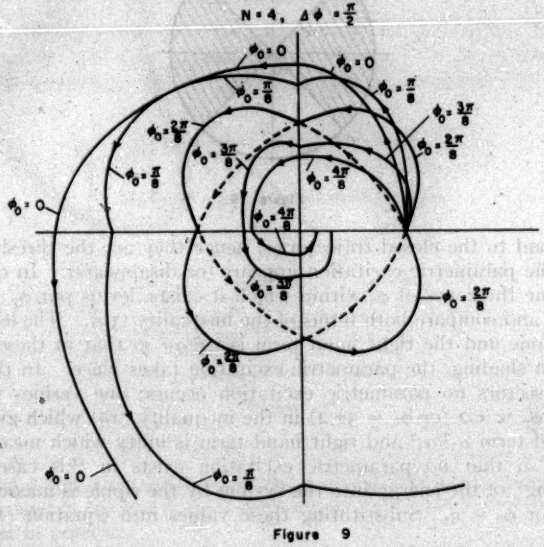

Figure 9

(3) *Third group* $N = 3, 5, 7, 9, \cdots$. Consider the case $\nu = 1$, $N = 3$; the conclusions are applicable to $N = 5, 7, \cdots$. We now have to use equation (15) which gives

$$a_3 = a_0 \frac{\alpha_1 f_1 f_3}{\alpha_2 f_2}. \tag{23}$$

The intervals are here: $\phi_0, \phi_1 = \phi_0 + (2\pi/3), \phi_2 = \phi_0 + (4\pi/3), \phi_3 = \phi_0$, and the condition for parametric excitation is

$$\frac{\alpha_1 f_1 f_0}{\alpha_2 f_2} \geqslant 1. \tag{24}$$

One sees at once that the closed trajectories correspond to the values $\phi_0 = 0$, $\phi_0 = \pi$, in which case $f_1 = f_2$ and

$$f_0 = \sqrt{\frac{\alpha_2^2 + \tan^2 \phi_0}{\alpha_1^2 + \tan^2 \phi_0}} = \frac{\alpha_2}{\alpha_1}$$

which gives $a_3 = a_0$ in equation (23). By symmetry there are four other values $\phi_0 = \pi/3$, $\phi_0 = 4\pi/3$, $\phi_0 = 2\pi/3$ and $\phi_0 = 5\pi/3$ for which

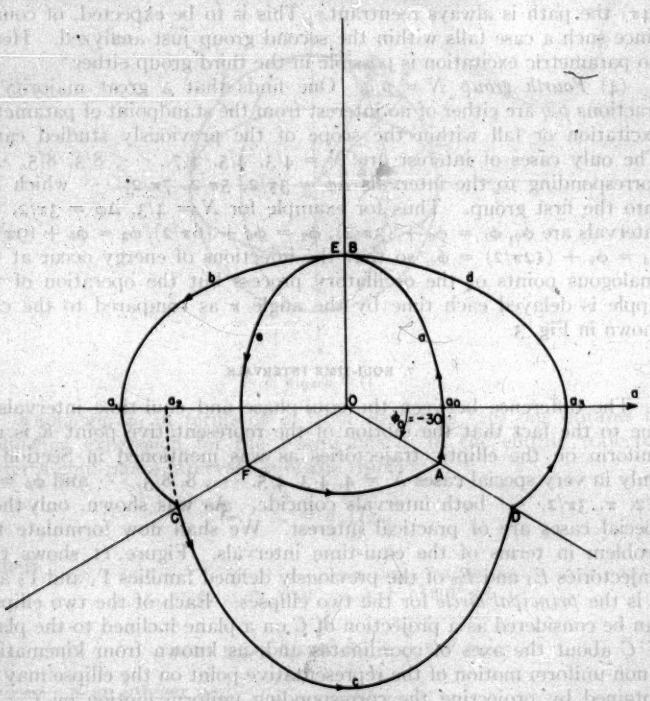

Figure 10

the trajectories close. It is to be noted, however, that although the trajectories close for these values of ϕ_0, the path is not reentrant so that for the second revolution $(2\pi, 4\pi)$ of the radius vector the representative point describes a trajectory different from that which it has followed during the first revolution $(0, 2\pi)$. Figure 10 shows a trajectory of this kind for $N = 3$, $\phi_0 = -30°$.

One ascertains this easily by comparing a_1, with a_4 which gives $a_1 = a_0(\alpha_1/\alpha_2)f_1$ and $a_4 = a_0(f_1f_3/f_2f_4)$. From the size of the intervals it is apparent that $_0f = f_3$, $f_1 = f_4$, so that

$$\frac{a_4}{a_1} = \frac{\alpha_2}{\alpha_1}\frac{f_0}{f_1f_2}.$$

In general $\alpha_1/\alpha_2 \neq f_0/f_1f_2$ although the equality may take place for special values of ϕ_0 which proves the statement. After two revolutions (4π) the path is always reentrant. This is to be expected, of course, since such a case falls within the second group just analyzed. Hence no parametric excitation is possible in the third group either.

(4) *Fourth group* $N = p/q$. One finds that a great majority of fractions p/q are either of no interest from the standpoint of parametric excitation or fall within the scope of the previously studied cases. The only cases of interest are $N = 4/3, 4/5, 4/7, \cdots, 8/3, 8/5, \cdots$, corresponding to the intervals $\Delta\phi = 3\pi/2, 5\pi/2, 7\pi/2, \cdots$ which fall into the first group. Thus for example for $N = 4/3$, $\Delta\phi = 3\pi/2$, the intervals are ϕ_0, $\phi_1 = \phi_0 + (3\pi/2)$, $\phi_2 = \phi_0 + (6\pi/2)$, $\phi_3 = \phi_0 + (9\pi/2)$, $\phi_4 = \phi_0 + (12\pi/2) = \phi_0$, so that the injections of energy occur at the analogous points of the oscillatory process but the operation of the ripple is delayed each time by the angle π as compared to the case shown in Fig. 3.

7. EQUI-TIME INTERVALS.

The difference between the equi-phase and equi-time intervals is due to the fact that the motion of the representative point R is not uniform on the elliptic trajectories as was mentioned in Section 5. Only in very special cases $N = 4, 4/3, 4/5, \cdots, 8, 8/3, \cdots$ and $\phi_0 = 0, \pi/2, \pi, 3\pi/2, \cdots$ both intervals coincide. As was shown, only these special cases are of practical interest. We shall now formulate the problem in terms of the equi-time intervals. Figure 11 shows two trajectories E_1 and E_2 of the previously defined families Γ_1 and Γ_2 and C is the *principal circle* for the two ellipses. Each of the two ellipses can be considered as a projection of C on a plane inclined to the plane of C about the axes of coordinates and, as known from kinematics, a non-uniform motion of the representative point on the ellipse may be obtained by projecting the corresponding uniform motion on C. In view of this to a position R of the representative point on E_1 defined by angle ϕ there corresponds a point $\bar{R}$ on C defined by the angle $\bar{\phi}$; one obtains a similar relation for the other ellipse E_2. Since in the phase plane representation the ratio of the semiaxes along the y and the x axes for the E_1 and E_2 ellipses is $\alpha_1^2 = 1 + \gamma$ and $\alpha_2^2 = 1 - \gamma$ respectively, one finds the relations connecting ϕ and $\bar{\phi}$ of the form

$$\tan\phi = \alpha_1^2 \tan\bar{\phi} \quad (\text{for } E_1); \qquad \tan\phi = \alpha_2^2 \tan\bar{\phi} \quad (\text{for } E_2). \quad (26)$$

Both angles ϕ and $\bar{\phi}$ coincide when the radius vector coincides with the coordinate axes. This is the reason why for the most interesting case of parametric excitation shown in Fig. 3 there is no difference between the two angles ϕ and $\bar{\phi}$. For the intermediate positions of the radius vector there exists in general a difference between the two intervals, the equi-phase ϕ and the equi-time $\bar{\phi}$. In this manner the equi-time intervals on the elliptic trajectories are reducible to the equi-phase angles $\bar{\phi}$ on the principal circle.

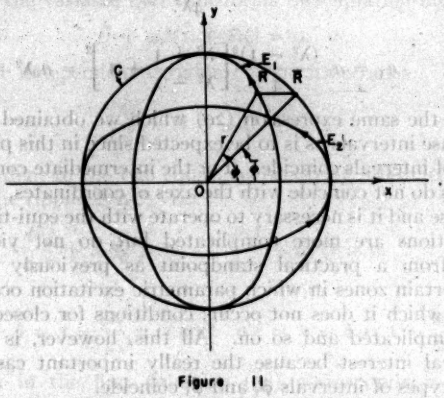

Figure 11

Replacing ϕ_i in terms of $\bar{\phi}_i$ from equations (26) one obtains the following expression for the x-semi-axis a_n of the elliptic trajectory after n equi-phase intervals $\bar{\phi}_i$ of the ripple

$$a_n = a_0 \prod_{i=1}^{i=n} g_n, \tag{27}$$

where

$$g_n = g_{2\nu-1} = \sqrt{\cos^2 \bar{\phi}_{2\nu-1} + \lambda^2 \sin^2 \bar{\phi}_{2\nu-1}};$$

$$g_n = g_{2\nu} = \sqrt{\cos^2 \bar{\phi}_{2\nu} + \frac{1}{\lambda^2} \sin^2 \bar{\phi}_{2\nu}}, \tag{28}$$

where ν is an integer and

$$\lambda^2 = \frac{\alpha_1^2}{\alpha_2^2} = \frac{1+\gamma}{1-\gamma} > 1.$$

It is apparent that $g_{2\nu-1} > 1$ and $g_{2\nu} < 1$. Expressions (28) can be also written in the form

$$g_{2\nu-1} = \frac{\sqrt{\lambda^2 - 1}}{\sqrt{2}} \sqrt{a - \cos \bar{\phi}_{2\nu-1}}; \qquad g_{2\nu} = \frac{\sqrt{\lambda^2 - 1}}{\sqrt{2\lambda}} \sqrt{a + \cos \bar{\phi}_{2\nu}}, \tag{29}$$

where

$$a = \frac{\lambda^2 + 1}{\lambda^2 - 1} > 1.$$

As an example let us apply equations (29) to the previously studied case $N = 2\nu = 4$. In terms of the equi-time intervals one has now: $2\bar{\phi}_1 = 2\bar{\phi}_0 + \pi, \cdots$ which gives

$$a_4 = a_0 g_1 g_2 g_3 g_4 = a_0 \frac{(\lambda^2 - 1)^2}{4\lambda^2} (a + \cos 2\bar{\phi}_0)^2.$$

For

$$\phi_0 = 0, \qquad a_4 = a_0 \frac{(\lambda^2 - 1)^2}{4} \left[\frac{\lambda^2 + 1}{\lambda^2 - 1} + 1 \right]^2 = a_0 \lambda^2 = a_0 \frac{\alpha_1^2}{\alpha_2^2},$$

which gives the same expression (20) which we obtained by means of the equi-phase intervals as is to be expected since in this particular case both types of intervals coincide. For the intermediate conditions when the intervals do not coincide with the axes of coordinates, this generally is not the case and it is necessary to operate with the equi-time intervals. The calculations are more complicated but do not yield anything interesting from a practical standpoint as previously investigated. There are certain zones in which parametric excitation occurs followed by zones in which it does not occur; conditions for closed trajectories are more complicated and so on. All this, however, is of relatively little practical interest because the really important cases are those where both types of intervals ϕ_i and $\bar{\phi}_i$ coincide.

8. PARAMETRIC EXCITATION OF A DISSIPATIVE CIRCUIT BY A CAPACITY RIPPLE.

Let us consider the problem of excitation of a dissipative circuit whose equation is

$$L_0 \ddot{q} + R_0 \dot{q} + \frac{1}{C_0} q = 0. \tag{30}$$

Dividing by L_0 and putting $1/L_0 C_0 = \omega_0^2$; $R_0/L_0 = 2p_0$ this equation can be written as

$$\ddot{q} + 2p_0 \dot{q} + \omega_0^2 q = 0. \tag{31}$$

Introducing a new variable Q defined by the equation

$$q = Qe^{-\int p dt} = Qe^{-\int p_0 dt} \tag{32}$$

equation (31) becomes

$$\ddot{Q} + \omega_2^2 Q = 0, \tag{33}$$

where

$$\omega_2^2 = \omega_0^2 - p^2 - \frac{dp}{dt} = \omega_0^2 - p_0^2 = \omega_1^2.$$

If one of the parameters L or C is a periodic function of time, ω_1^2 is also periodic and equation (33) is generally of the Hill type. With our

assumption of a rectangular ripple it is of the Hill-Meissner type which we will consider. Replacing in equation (30) C_0 by

$$C_0 \pm \Delta C = C_0\left(1 \pm \frac{\Delta C}{C_0}\right) = C_0(1 \pm \gamma_c),$$

the equation (31) becomes

$$\ddot{q} + 2p_0\dot{q} + \omega_0^2(1 \mp \gamma_c)q = 0. \tag{34}$$

The change of the variable (32) transforms this equation into the form

$$\ddot{Q} + \omega_1^2(1 \mp \delta)Q = 0. \tag{35}$$

Introducing the independent variable $\tau = \omega_1 t$ it becomes

$$\frac{d^2Q}{d\tau^2} + (1 \mp \delta)Q = 0 \tag{36}$$

which is equivalent to the two equations occurring alternatively at the frequency of the ripple $\pm \Delta C$, viz.:

$$\frac{d^2Q}{d\tau^2} + \alpha_1^2 Q = 0; \qquad \frac{d^2Q}{d\tau^2} + \alpha_2^2 Q = 0, \tag{37}$$

where

$$\alpha_1^2 = 1 + \delta = 1 + \frac{\omega_0^2}{\omega_1^2}\gamma_c \quad \text{and} \quad \alpha_2^2 = 1 - \delta = 1 - \frac{\omega_0^2}{\omega_1^2}\gamma_c. \tag{38}$$

The plus sign in the first equation (38) corresponds to the value $C = C_0 - \Delta C$. Equations (37) have the same forms as equations (10) so that the previous conclusions are applicable here with the difference however, that equations (37) contain the dependent variable Q whereas the equations (10) contain the variable q, the two variables being related by equation (32).

The trajectories of equation (37) as previously, are either convergent or divergent piece-wise analytic spirals, formed by elliptic arcs, the closed trajectories appearing as a threshold between the two forms of spirals. For a closed trajectory, clearly, Q is bounded, hence q is monotone decreasing. This means that to the closed trajectories in the $(Q, dQ/d\tau)$ plane correspond convergent spirals in the $(q, dq/d\tau)$ plane, so that no parametric excitation is possible in this case. It is obvious that for a parametric excitation the amplitudes of q must be either monotone increasing or, at least, constant, which requires

$$Q = Q_0 e^{+\int p_1 dt}, \tag{39}$$

where $p_1 \geqslant p_0$. This means that the trajectories in the $(Q, dQ/d\tau)$ plane must be divergent spirals with negative decrement * $-p_1$ having the aboslute value p_1 greater than, or, at least, equal to the positive

* Negative decrement, or increment, because the spirals are divergent.

decrement $p_0 = R_0/2L_0$ of the dissipatory circuit (R_0, L_0, C_0). Physically this means that the energy injections into the circuit by the ripple $\pm \Delta C$ must on the average be greater than the energy dissipation. This condition is obtained by transforming the differential equation of the dissipative circuit

$$L_0\ddot{q} + R_0\dot{q} + \frac{1}{C_0}q = 0$$

into the form

$$\frac{d^2q}{d\tau^2} + 2P_0\frac{dq}{d\tau} + q = 0, \tag{40}$$

where $\tau = \omega_0 t$; $\omega_0 = 1/\sqrt{L_0C_0}$ and $P_0 = R_0/2L_0\omega_0$. As is well known the trajectories of equation (40) are convergent logarithmic spirals and the ratio of the amplitudes $q_{2\pi}$ and q_0 after one period is

$$\frac{q_{2\pi}}{q_0} = e^{-P_0 2\pi}, \tag{41}$$

On the other hand in the optimum case ($N = 4$, $\phi_0 = 0$) of the parametric excitation it follows from equation (20) that for the divergent spirals this ratio is

$$\frac{q_{2\pi}}{q_0} = \frac{\alpha_1^2}{\alpha_2^2} = e^{+P_1 2\pi}, \tag{42}$$

which defines the average increment

$$P_1 = \frac{1}{2\pi}\log\frac{\alpha_1^2}{\alpha_2^2}. \tag{43}$$

Expressing the condition for the parametric excitation $P_1 \geqslant P_0$ and substituting for α_1^2, α_2^2 and P_0 their values one gets

$$\gamma_c \geqslant \frac{\omega_0^2 - p_0^2}{\omega_0^2}\frac{e^{\mu_0} - 1}{e^{\mu_0} + 1} = \gamma_c', \tag{44}$$

where $\mu_0 = \pi R_0/L_0\omega_0$. Since α_1^2 and α_2^2 are positive and $\alpha_1^2 > \alpha_2^2$ one obtains the other limit for γ_c by expressing that α_2^2 is positive. This gives

$$\gamma_c \leqslant \frac{\omega_0^2 - p_0^2}{\omega_0^2} = \gamma_c''. \tag{45}$$

From equations (44) and (45) it follows that γ_c must be in the interval (γ_c', γ_c'') viz.

$$\gamma_c' < \gamma_c < \gamma_c'' \tag{46}$$

in order to obtain the parametric excitation. For $R_0 = 0$, $\mu_0 = 0$, $p_0 = 0$ hence the interval is (0, 1) and decreases with R_0 increasing. For $\omega_0^2 - p_0^2 = \omega_1^2 = 0$, that is for $R_0 = 2\sqrt{L_0/C_0}$ the interval (γ_c', γ_c'')

reduces to zero with $\gamma_c' = \gamma_c'' = 0$. One ascertains that this is the condition for critical damping. It is, therefore, impossible to obtain a parametric excitation of a critically damped, or over-damped circuit.

9. PARAMETRIC EXCITATION OF A DISSIPATIVE CIRCUIT BY AN INDUCTANCE RIPPLE.

In Section 3, it was mentioned that for a non-dissipative circuit the effects of the capacity variation ($\pm \Delta C$) and of the inductance variation ($\pm \Delta L$) are the same from the standpoint of the excitation. This is due to the fact both L and C enter symmetrically into the expression $\omega_0^2 = 1/L_0C_0$ for the frequency. For a dissipatory circuit the situation is different in that the capacity enters only into the expression for the damped frequency $\omega_1^2 = \omega_0^2 - p_0^2$ (through ω_0^2) but does not appear in the decrement $p_0 = R_0/2L_0$. As regards the inductance, it appears both in the expressions for the frequency and for the decrement. A priori, one may expect different results in both cases.

We shall now consider the effect of a step-wise variation $\pm \Delta L$ of the coefficient of inductance L and will define the modulation index $\gamma_L = \Delta L/L_0$. The difference with the preceding case is in that the decrement

$$p = \frac{R_0}{2L} = \frac{R_0}{2L_0(1 \pm \gamma_L)}$$

is not constant now but also undergoes a modulation. Moreover, in the transformation (32) $p \neq p_0$ so that in equation (33) the value of ω_2^2 is now *

$$\omega_2^2 = \omega_0^2 - p^2 - \frac{dp}{dt}. \tag{47}$$

For a step-wise variations $\pm \Delta L$, the derivative $dp/dt = 0$ throughout except at the points at which the jumps $\pm \Delta L$ take place; at these points dp/dt has no meaning. It is possible, however, to obviate this difficulty by surrounding the jumps in the (L, t) plane by the infinitesimal intervals $\pm \epsilon$ parallel to the L axis. Since the trajectories are continuous curves, although not analytic at the points at which the jumps occur, one can disregard what happens in these intervals. The expression (47) is then

$$\begin{aligned}\omega_2^2 &= \omega_0^2(1 \mp \gamma_L) - p_0^2(1 \mp 2\gamma_L) \\ &= (\omega_0^2 - p_0^2) \mp \gamma_L(\omega_0^2 - 2p_0^2) = \omega_1^2 \mp \gamma_L(\omega_1^2 - p_0^2)\end{aligned} \tag{48}$$

neglecting the term with γ_L^2. The corresponding Hill-Meissner equation is here

$$\ddot{Q} + \omega_1^2\left[1 \mp \gamma_L \frac{\omega_1^2 - p_0^2}{\omega_1^2}\right]Q = 0. \tag{49}$$

* We will retain the notation $\omega_1^2 = \omega_0^2 - p_0^2$. In Section 8 we had $\omega_2^2 \equiv \omega_1^2$. Here $\omega_2^2 = \omega_0^2 - p^2$, where $p \neq p_0$.

Introducing the "angular time" $\tau = \omega_1 t$ this equation becomes

$$\frac{d^2Q}{d\tau^2} + \alpha_1^2 Q = 0; \qquad \frac{d^2Q}{d\tau^2} + \alpha_2^2 Q = 0, \tag{50}$$

where

$$\alpha_1^2 = 1 + \gamma_L \frac{\omega_1^2 - p_0^2}{\omega_1^2} \quad \text{and} \quad \alpha_2^2 = 1 - \gamma_L \frac{\omega_1^2 - p_0^2}{\omega_1^2}.$$

Moreover $\alpha_1^2 > \alpha_2^2$ equations (50) have the same form as equation (37) but the values of α_1^2 and α_2^2 are different. Using equation (43) and expressing that $P_1 \geqslant P_0$ we obtain

$$\gamma_L \geqslant \frac{\omega_1^2}{\omega_1^2 - p_0^2} \frac{e^{\mu_0} - 1}{e^{\mu_0} + 1} = \gamma_L' \tag{51}$$

in the previous notations.

It is easy to show that the second limit γ_L'' does not exist here. In fact, expressing the condition that $\alpha_2^2 > 0$ one finds

$$\gamma_L \leqslant \frac{\omega_1^2}{\omega_1^2 - p_0^2} = \frac{\omega_0^2 - p_0^2}{\omega_0^2 - 2p_0^2}.$$

On the other hand it has been assumed that $\gamma_L \ll 1$ so that the preceding inequality does not give any useful information. Hence in the case of the excitation by the inductance ripple only one condition (51) exists instead of the two as previously.

It is to be noted that in all preceding discussions, it has been assumed that the modulation index γ_c or γ_L is small which permits writing $1/(1 \pm \gamma) \leqq 1 \mp \gamma$. If one waives this condition more terms in the expansion of $1/(1 \pm \gamma)$, $1/(1 \pm \gamma)^2$, etc., must be retained which leads to more complicated expressions.

10. APPROACH TO THE MATHIEU EQUATION.

The preceding graphical method does not applly to the Mathieu equation in which case instead of the two elliptic families Γ_1 and Γ_2 there exists an infinity of such families varying continuously between these two limiting families. One may conceive that the representative moves continuously from one family to the other so that at the limit it has only one point on a curve of each family. In spite of the difficulty of representing such a motion in the phase plane a certain approach to this condition can be made by approximating the continuous variation of the parameter along a sinusoidal curve instead of a rectangular ripple by a step-wise ladder function following the curve in the manner shown in Fig. 12. It is clear that if one makes the intervals $\Delta\phi$ sufficiently small there will be a discrete sequence of families $\Gamma_1, \Gamma_2, \cdots, \Gamma_n$ of elliptic trajectories on which the representative point will move during each interval $\Delta\phi_i$ passing from a curve of one family, say, Γ_i to the

following Γ_{i+1} by continuity. It is apparent also as the size of the intervals $\Delta\phi = 2\pi/n$ decreases the motion will approach the motion which is obtained when the parameter varies continuously as in the case of the Mathieu equation.

If one applies the reasoning of Section 7, one ascertains easily that the equations (28) still hold provided we define the quantities λ^2 by the relation

$$\lambda^2_{i,i+1} = \frac{\alpha_i^2}{\alpha^2_{i+1}} = \frac{1 + \gamma_0 \sin \phi_i}{1 + \gamma_0 \sin (\phi_i + \Delta\phi)} \leqq 1 - \gamma_0 \cos \phi_i \Delta\phi \qquad (53)$$

assuming that $\gamma_0 \ll 1$ and $\Delta\phi$ very small.

Introducing the factor β taking into account the relative frequency of the parameter variation and an arbitrary phase angle ψ the general

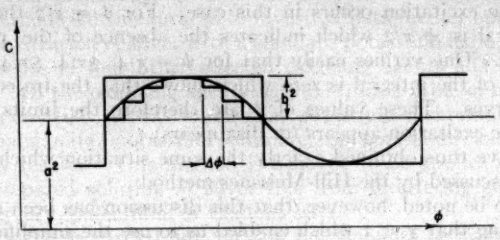

Figure 12

expression for this coefficient is

$$\lambda^2_{i,i+1} = 1 - \gamma_0 \cos \beta(\phi_i - \psi)\Delta\phi. \qquad (54)$$

The expression for the function g_i (compare with equation 28) becomes

$$g_i = \sqrt{1 - \gamma_0 \cos \beta(\phi_i - \psi) \sin^2 \phi_i \Delta\phi}$$

and the condition for parametric excitation is

$$\prod_{i=1}^{i=n} g_i = \prod_{i=1}^{i=n} \sqrt{1 - \gamma_0 \cos \beta(\phi_i - \psi) \sin^2 \phi_i \Delta\phi} \geqslant 1. \qquad (55)$$

This condition is clearly equivalent to the following

$$I(\psi) = \sum_{i=1}^{i=n} \log (1 - \gamma_0 \cos \beta(\phi_i - \psi) \sin^2 \phi_i \Delta\phi) \geqslant 0. \qquad (56)$$

Introducing the notation $f(\phi_i) = \cos \beta(\phi_i - \psi) \sin^2 \phi_i$ it is obvious that

$$-\gamma_0 f(\phi_i)\Delta\phi < \log (1 - \gamma_0 f(\phi_i)\Delta\phi) < -\gamma_0 f(\phi_i)\Delta\phi + \frac{\gamma_0^2}{2} f^2(\phi_i)\Delta\phi^2. \qquad (57)$$

By letting the index i run through the values $1, 2, \cdots, n$ the sum of

the left hand terms of (57) clearly converges to the limit

$$J_1(\psi) = -\gamma_0 \int_0^{2\pi} f(\phi) d\phi. \tag{58}$$

The sum of the right hand terms of (57) converges to the same limit since the limit of the series $\sum_{i=1}^{n} f^2(\phi_i)\Delta\phi^2$ for $n = \infty$ is zero. Hence the limit of the sum (56) exists and is given by equation (58). The condition for parametric excitation for a continuous variation of the parameter is then

$$J(\psi) = \int_0^{2\pi} \cos\beta(\phi - \psi) \sin^2 \phi d\phi \leqslant 0. \tag{59}$$

As an example consider the previously investigated case ($N = 4, \beta = 2$). For $\psi = 0$ the value of the integral (59) is $-\pi/2$ which means that parametric excitation occurs in this case. For $\psi = \pi/2$ the value of the integral is $+\pi/2$ which indicates the absence of the parametric excitation. One verifies easily that for $\psi = \pi/4, 3\pi/4, 5\pi/4$ and $7\pi/4$ the value of the integral is zero which shows that the trajectories are closed curves. These values of ψ are therefore the limits at which parametric excitation appears (or disappears).

We have thus obtained exactly the same situation which has been already discussed by the Hill-Meissner method.

It is to be noted, however, that this discussion has been conducted by assuming that $\gamma \ll 1$ which enabled us to use the simplified expression (53) for the coefficient $\lambda^2_{i,i+1}$. By waiving this restriction calculations are more complicated but the qualitative picture of the phenomenon remains substantially the same.

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THE TITRATION OF SILVER WITH POTASSIUM IODIDE.

USE OF FOUR SELECTED OXIDIZING AGENTS AND STARCH AS INDICATORS.

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A method of determining silver by titration with standard potassium iodide in the presence of starch and ceric ions has been described (1, 2, 4, 8, 9). The advantages of this method over the Gay-Lussac and Volhard methods led the authors to investigate its further application; namely, the titration of silver with a standard potassium iodide solution in the presence of four other selected oxidizing agents and starch as indicators. Of the oxidizing agents selected from a table of oxidation-reduction potentials, it was found that potassium dichromate, potassium permanganate, potassium bromate, or potassium iodate gave satisfactory results. This method is believed to be applicable to any properly prepared solution of silver free from interfering substances, such as mercury, palladium, or thallous salts.

Titrations were carried out varying the amounts of reagents so as to establish the best conditions for the greatest degree of accuracy. Results were obtained by titrating in the presence of elements sometimes associated with silver to test for possible interference.

REAGENTS.

All chemicals were C.P. analytical grade. Silver nitrate and potassium iodide were twice recrystallized and powdered. The silver nitrate was dried at 120° C. for one hour and the potassium iodide was dried at 180° C. for one hour. Standard 0.1 N solutions of silver nitrate and potassium iodide were prepared by weight.

Potassium dichromate, potassium permanganate, potassium bromate, and potassium iodate were recrystallized and dried in a desiccator over calcium chloride. Solutions 0.1 N as oxidizing agents were prepared from these salts.

Salts of arsenic, bismuth, cobalt, copper, iron, lead, manganese, mercury, nickel, and zinc were used without purification.

The starch solution was prepared by dissolving 0.5 gram of soluble starch in 5 ml. of cold water. This solution was poured into 95 ml. of boiling water and boiled for a few minutes. The 0.5 per cent. starch solution was kept in a glass stoppered bottle.

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PROCEDURE.

Weigh a sample containing between 1 to 2 grams of silver into a 400 ml. beaker and add 10 ml. of nitric acid (1:1). Heat the beaker until the sample is dissolved and add 20 ml. of distilled water. Continue the heating until the oxides of nitrogen are removed as shown by the disappearance of the brown color. Transfer this solution to a 250 ml. volumetric flask and dilute to the mark with distilled water.

Pipet a 25 ml. aliquot of the dilution into a 250 ml. Erlenmeyer flask containing 35 ml. of approximately 6 N sulfuric acid. Dilute the solution to approximately 100 ml. with distilled water.

After the addition of 1 drop of 0.1 N potassium permanganate, or 3 drops of 0.1 N potassium dichromate, potassium bromate, or potassium iodate, add 3 ml. of freshly prepared 0.5 per cent. starch solution. Titrate with the standard 0.1 N potassium iodide solution to the first permanent blue-green color. Run a blank determination under identical conditions but omit the sample.

EXPERIMENTAL.

The conditions necessary for the highest accuracy were first determined. Sufficient pure, dry potassium iodide to give a 0.1 N solution was weighed, transferred to a liter volumetric flask, and diluted to the mark. Solutions of known strength of pure silver nitrate were similarly prepared. Accurately measured volumes of the silver nitrate solution were transferred to 250 ml. Erlenmeyer flasks. Increasing amounts of sulfuric acid were added and the solutions diluted to 100 ml. with distilled water. The amounts of oxidizing agent and starch recommended in the procedure were added and the solutions titrated.

In a similar manner the amounts of the oxidizing agents and the starch were varied until the optimum conditions were determined.

Using the best conditions determined as above, increasing amounts of different elements were added to the solution to be titrated. In this manner, the tests for possible interference were made.

Three alloys were analyzed for silver by the Gay-Lussac method, the Volhard method, and the method presented here. The procedure used for the Gay-Lussac method is given by Kolthoff and Furman (6). The procedure used for the Volhard method is that given by Hillebrand and Lundell (5). Potassium permanganate was used as the oxidizing agent in the potassium iodide method with alloy number one that contained silver, copper, and zinc. Potassium dichromate was used as the oxidizing agent with alloy number two that contained silver and copper. Potassium bromate was the oxidizing agent used with alloy number three that contained silver, copper, and zinc.

An additional comparison of the accuracy of the method was obtained by using ceric ammonium sulfate as the oxidizing agent in the procedure described by Bloom and McNabb (1).

DISCUSSION.

One of the advantages of the method presented here is the choice of one of four oxidizing agents and starch as indicators in the titration of silver with a standard potassium iodide solution. Another advantage is the use of a standard potassium iodide solution prepared by weight, making a standardization unnecessary. The chief advantage is the sharp and easily detected end-point which eliminates one source of uncertainty found in the Gay-Lussac and Volhard methods.

Mulder (5) reported that many elements interfered in the Gay-Lussac method. Mercury was the most harmful. Bismuth readily hydrolyzed to render the end-point indistinct.

When Dewey (3) studied the effect of various elements on the Volhard method he found several that interfered. Mercury salts interfered because they formed slightly dissociated mercury thiocyanate. Copper salts formed dark colored copper thiocyanate that obscured the end-point. Large quantities of nickel and cobalt interfered because their color made the end-point difficult to detect.

When using the potassium iodide method presented here no appreciable difference was obtained when 100 to 200 milligrams of silver were titrated in the presence of separate solutions of arsenic, bismuth, cobalt, copper, iron, lead, manganese, nickel, and zinc. These various elements were present in concentrations ranging from 4 to 300 milligrams.

Copper salts might have been expected to interfere in this procedure due to reaction with potassium iodide to form cuprous iodide and iodine. Iron might also have been expected to interfere because ferric ions oxidize potassium iodide in acid solutions. The oxidation-reduction potentials of any of the oxidizing agents, however, are much stronger than those of cupric or ferric ions. A small amount of potassium iodide, therefore, would be oxidized by the added oxidizing agent before reaction with cupric or ferric ions could occur, and there would be no interference.

Lead often found with silver does not interfere because of the insolubility of lead sulfate.

Increasing the amounts of sulfuric acid, oxidizing agents, or starch indicated that with very large amounts of these reagents high results were obtained. When using amounts recommended in the procedure, satisfactory values resulted.

Attempts to substitute nitric acid or acetic acid for the sulfuric acid were not successful.

Analytical results given in Tables I to IV show the accuracy of the method. These may be compared with results obtained using the ceric ammonium sulfate procedure which are shown in Table V.

In Table VI results are given for the analyses of three alloys in which the Gay-Lussac and Volhard procedures were selected for comparison.

TABLE I.

Potassium Dichromate and Starch as Indicators.

Ag Taken, Gram	Ag Found, Gram	Difference, Gram
0.0539	0.0541	+0.0002
0.0539	0.0540	+0.0001
0.0539	0.0541	+0.0002
Ag Taken, Gram	**Ag Found, Gram**	**Difference, Gram**
0.1079	0.1081	+0.0002
0.1079	0.1082	+0.0003
0.1079	0.1081	+0.0002
Ag Taken, Gram	**Ag Found, Gram**	**Difference, Gram**
0.2158	0.2162	+0.0004
0.2158	0.2163	+0.0005
0.2158	0.2161	+0.0003

TABLE II.

Potassium Permanganate and Starch as Indicators.

Ag Taken, Gram	Ag Found, Gram	Difference, Gram
0.0539	0.0540	+0.0001
0.0539	0.0539	0.0000
0.0539	0.0540	+0.0001
Ag Taken, Gram	**Ag Found, Gram**	**Difference, Gram**
0.1079	0.1082	+0.0003
0.1079	0.1082	+0.0003
0.1079	0.1081	+0.0002
Ag Taken, Gram	**Ag Found, Gram**	**Difference, Gram**
0.2158	0.2161	+0.0003
0.2158	0.2162	+0.0004
0.2158	0.2163	+0.0005

TABLE III.

Potassium Bromate and Starch as Indicators.

Ag Taken, Gram	Ag Found, Gram	Difference, Gram
0.0539	0.0541	+0.0002
0.0539	0.0541	+0.0002
0.0539	0.0540	+0.0001
Ag Taken, Gram	**Ag Found, Gram**	**Difference, Gram**
0.1079	0.1081	+0.0002
0.1079	0.1082	+0.0003
0.1079	0.1081	+0.0002
Ag Taken, Gram	**Ag Found, Gram**	**Difference, Gram**
0.2158	0.2162	+0.0004
0.2158	0.2162	+0.0004
0.2158	0.2163	+0.0005

TABLE IV.

Potassium Iodate and Starch as Indicators.

Ag Taken, Gram	Ag Found, Gram	Difference, Gram
0.0539	0.0540	+0.0001
0.0539	0.0541	+0.0002
0.0539	0.0542	+0.0003
Ag Taken, Gram	**Ag Found, Gram**	**Difference, Gram**
0.1079	0.1082	+0.0003
0.1079	0.1081	+0.0002
0.1079	0.1081	+0.0002
Ag Taken, Gram	**Ag Found, Gram**	**Difference, Gram**
0.2158	0.2162	+0.0004
0.2158	0.2163	+0.0005
0.2158	0.2162	+0.0004

TABLE V.

Ceric Ammonium Sulfate and Starch as Indicators.

Ag Taken, Gram	Ag Found, Gram	Difference, Gram
0.0539	0.0540	+0.0001
0.0539	0.0541	+0.0002
0.0539	0.0540	+0.0001
Ag Taken, Gram	**Ag Found, Gram**	**Difference, Gram**
0.1079	0.1081	+0.0002
0.1079	0.1082	+0.0003
0.1079	0.1081	+0.0002
Ag Taken, Gram	**Ag Found, Gram**	**Difference, Gram**
0.2158	0.2162	+0.0004
0.2158	0.2163	+0.0005
0.2158	0.2161	+0.0003

TABLE VI.

Comparison of Methods.

Alloy—(1)	Method Used	Per Cent. Silver
(Ag, Cu, Zn)	Gay-Lussac	50.23
		50.21
	Volhard	50.31
		50.31
	Potassium Iodide	50.34
		50.34
Alloy—(2)		
(Ag, Cu)	Gay-Lussac	89.75
		89.72
	Volhard	89.81
		89.90
	Potassium Iodide	89.89
		89.90

TABLE VI.—*Continued.*

Alloy—(3)	Method Used	Per Cent. Silver
(Ag, Cu, Zn)	Gay-Lussac	30.82
		30.82
	Volhard	30.85
		30.85
	Potassium Iodide	30.89
		30.89

SUMMARY.

A method was investigated for the volumetric determination of silver by titration with potassium iodide in the presence of starch and four selected oxidizing agents as indicators. The results obtained by this method show satisfactory levels of precision and accuracy, and are substantially identical with results by the Gay-Lussac and Volhard methods.

The presence of the following metals does not interfere in the method: arsenic, bismuth, cobalt, copper, iron, lead, manganese, nickel, and zinc. Metals such as mercury and palladium that form insoluble iodides must be removed before titrating the silver.

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NOTES FROM THE NATIONAL BUREAU OF STANDARDS.*

EXTRACTION OF ALUMINA FROM CLAY.

In the early days of the war, shipping losses cut imports of bauxite, the principal aluminum-bearing ore, to a point that seriously endangered the aluminum industry. Funds were made available by the War Department for the construction of a pilot plant at the Bureau to extract alumina from abundant American clays. Difficulty in obtaining equipment hampered construction, but by May 1943 a small plant capable of producing about 50 lbs. of alumina a day had been built. Since then the plant has been operated and improved, but certain phases, such as calcination of the oxide and recovery of acid, require further work.

The process consists in (1) roasting clay at about 700° C. to render the alumina soluble, (2) leaching the roasted product with dilute hydrochloric acid, (3) filtering to separate the insoluble siliceous matter from the solution containing the aluminum and soluble impurities such as iron and alkali salts, (4) concentrating the solution, (5) precipitating the aluminum as the hydrated chloride from the concentrated solution, (6) removing the crystals of hydrated aluminum chloride by centrifuging in a rubber-coated centrifuge, (7) washing the crystals to remove adhering impurities, (8) calcining the hydrated chloride to obtain alumina and to expel combined hydrochloric acid which is used in the next cycle, and (9) recovering hydrochloric acid from the waste products at the end of the process.

The alumina obtained in this pilot plant has an average purity of about 99.6 per cent., the significant impurities being 0.25 per cent. of chlorine, 0.02 per cent. of iron, and 0.05 per cent. of silicon. This compares favorably with alumina produced from high-grade bauxite ores. For some uses, the alumina possesses exceptional properties, for example, as a polishing material for metallographic specimens and as the raw component of certain heat-resisting enamels.

No recent cost estimate has been made, but about one year ago it was indicated that alumina made by this process would be approximately twice as expensive as the regular commercial product. However, reduction in price of hydrochloric acid when bought in large quantities, and improvement in plant operation should materially reduce this figure if a shortage of bauxite ore ever again threatens the industry.

* Communicated by the Director.

STRESS-CORROSION OF MAGNESIUM-BASE ALLOYS.

The susceptibility to stress corrosion of the popular magnesium-base alloys (about 95 per cent. magnesium, balance chiefly aluminum and zinc) is a matter of extreme interest to users. Surface treatments have been developed that render these alloys practically immune to ordinary corrosive attack, provided the specimen is not stressed; but the simultaneous action of stress and corrosion materially shortens the life of the specimen. Stress-corrosion failures have been observed in such mildly corrosive media as atmospheric exposure in semi-industrial areas; they are more pronounced in specimens exposed to marine atmospheres or immersed in sea water. Laboratory tests have shown that the unstressed alloys last indefinitely when immersed in an aqueous solution of 2 per cent. potassium chromate plus 3 per cent. sodium chloride, but that stressed specimens fail in a manner similar to the stress-corrosion failures in other media, and in a matter of minutes or even seconds if the stress is high enough. Results of these laboratory tests are expressed in terms of the "critical stress," i.e., the stress that causes the specimen to fail within 48 hours, immersed in the test solution. The critical stress, in general, appears to be at least 70 per cent. of the yield strength and, for some of the more resistant compositions, it may be as high as 95 per cent.

FIRST TWO SPECTRA OF COLUMBIUM.

John Winthrop (1606–1676), first governor of Connecticut, took pleasure in collecting minerals. Near his home in New London, he found a black rock (now known as columbite) which his grandson sent to London where it was placed in the British Museum. In 1801 Charles Hatchett (1765–1847), an English chemist, fused the ore with potassium carbonate. When he took up the melt with boiling water, a brown residue resulted. When nitric acid was added to the yellow filtrate, a copious white precipitate was thrown down. "The preceeding experiments shew," said he, "that the ore which has been analysed consists of iron combined with an unknown substance." Hatchell named the new element *Columbium*, but he did not succeed in reducing the compound to metal.

Columbium, as a shining steel-gray metal, was first prepared in 1864, but for more than 6 decades the few tiny specimens available were regarded as scientific curiosities. However, in 1929, C. W. Balke of Chicago exhibited before the American Chemical Society, large sheets and rods of this rare metal, and industrial uses of columbium began immediately. Recently, the addition of columbium and other metals to iron has resulted in revolutionary types of steel alloys which are now so complex that chemical analyses for composition control

during production can be efficiently made only by spectrographic methods.

Between 1895 and 1910 several attempts were made to describe the emission spectra characteristic of columbium. Twenty years ago, these early data were examined in the Bureau's Spectroscopy Section. As a result, the first regularities in columbium spectra were discovered, and it was concluded that improved descriptions of the spectra were desirable. From time to time new observations were made on wavelengths, relative intensities, ionization stages, and the behavior of spectral lines in a magnetic field (Zeeman effect). The structural analyses of the first two spectra of columbium were prosecuted until 95 per cent. of the total observed intensity was accounted for by identified atomic energy changes.

In order to facilitate spectrochemical analyses of alloys containing columbium, the best information now available for the first two spectra has been prepared for publication by Curtis J. Humphreys and William F. Meggers, and will appear as RP1656 in the June number of the *Journal of Research*. The total number of lines definitely characteristic of neutral columbium atoms is 3313, of which 2836 are interpreted as combinations of 364 identified atomic energy states. The number of lines ascribed to singly ionized columbium atoms is 1723, of which 1494 are explained as transitions between 183 ionic energy levels.

INFLUENCE OF VARIOUS CURING METHODS ON CONCRETE.

Comparative strength and wear tests of portland cement concrete are reported in an article prepared by H. C. Volmer for publication in the Proceedings of the Highway Research Board. The concrete specimens were stored at 70° F. and 50–60 per cent. relative humidity. Six methods of curing—burlap, integral and surface calcium chloride, and 3 samples of sprayed surface membranes—were used. Damp burlap for 18 hours, with calcium chloride used either integrally or spread on the surface upon removal of the burlap, gave 28-day strengths of the same order as the 3-day wet burlap method. The strengths obtained with the 3 sprayed membranes were lower. Compared to the specimens cured with burlap for 3 days, the specimens cured with surface or integral calcium chloride had higher wear resistance, and specimens cured with the three membranes had lower wear resistance.

STABILITY OF LUBRICATING GREASES.

Greases deteriorate in service primarily because of (a) breakdown of the grease structure, (b) absorption of water, and (c) change in composition resulting from oil evaporation and from oxidation.

Investigations at the Bureau on working of greases have shown that they differ markedly as regards the rate of breakdown of the grease structure, dependent upon the characteristics of the oil and soap used in their manufacture, but particularly upon the percentage of soap used. No grease has been tested thus far that does not continue to break down with increased working. However, it has been found possible to produce greases in which this breakdown takes place so slowly that it is not serious. This is true of very few commercial greases.

Greases also have been found to differ greatly as regards the effect of water, depending upon the nature of the soap used in their manufacture. New test methods have been developed for water resistance of greases, and also for loss of oil by evaporation at elevated temperatures.

These conclusions—the results of an intensified study of greases for military needs—have led to renewed activity on the part of manufacturers; this can hardly fail to result in marked improvement in the greases available for civilian use.

REVISED SIMPLIFIED PRACTICE RECOMMENDATION FOR FILES AND RASPS.

The proposed revision of Simplified Practice Recommendation R6–44, Files and Rasps (American Pattern, and Straight- and Curved-tooth Milled Files), having been accorded the required degree of acceptance by manufacturers, distributors and users, became effective on May 1, 1945.

The revision makes twelve departures from the superseded issue. The width and thickness dimensions of some of the sizes and types, which had been reduced as a conservation measure, have been restored; several sizes of flat brass files and hand-saw blunt files have been added, also second-cut in five sizes of mill files. One size in each of six other types is eliminated.

Until printed copies are ready, a limited number of mimeographed copies are available free of charge on addressing the Division of Simplified Practice, National Bureau of Standards, Washington 25, D. C.

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Adapter Plug Provides Mobility for Field Switchboards.—(*Electrical Engineering*, Vol. 63, No. 9.) A new light compact telephone switching unit developed by the Signal Corps, Army Service Forces, is supplanting for emergency field service the bulky switchboards hitherto used. This unit, the Adapter Plug U-4/GT, can be used for terminating a local battery telephone line, and several plugs, used with a field telephone for the operator's set, will provide a workable switchboard substitute which may be set up rapidly.

Weighing only one ounce, the plug can be transported to many spots inaccessible to a battery switchboard. It also is moistureproof which makes it invaluable for jungle or mountain operations, for beach landing in rain-drenched areas, and for air-borne operations behind the enemy lines. The transparent plastic body of the plug contains fasteners for line connections, a neon lamp that responds to ringing signals, and two combination jacks and plugs for tandem connections between adapter plugs. The resistance and neon lamps are inside the plastic body which makes them virtually impervious to moisture and fungus attacks. A luminescent identification plate on which the operator may inscribe the line number is embedded in the plastic.

The plug builds up a flexible switchboard upon which the operator can make not only individual connections but also conference connections by calling individually the parties desired and connecting the adapter plugs in tandem.

The U-4/GT was developed in response to an elementary need, realized especially in the Pacific war theater, of substituting a visual signal for the bell signal. The latter could be heard from considerable distances by enemy snipers. Extension of the use of the plugs to the building of substitute emergency switchboards was a by-product. The visual signal functions when the operator attaches the field-wire pair entering his outpost to an adapter plug and connects his telephone by a short length of field wire to another adapter plug. Incoming calls flash the lamp in the first plug.

R. H. O.

Calculator Widens Scope of Practical Mathematics.—(*Electrical Engineering*, Vol. 63, No. 9.) A versatile calculating machine recently was presented by International Business Machines Corporation to Harvard University to be used by the United States Navy for the duration. Employing a unique automatic sequence control which receives problems on a coded tape, the machine will solve practically any known problem to 23 significant figures, including problems involving computation and tabulation of functions, evaluation of integrals, solution of simultaneous linear algebraic equations, and harmonic and statistical analysis. Its basic performance comprises addition, subtraction, use of parentheses and brackets, multiplication, division, interpolation or the location of values of functions in mechanical tables of mathematical functions, card punching of computations, card feeding, and storage of numerical data. Particularly benefited by such calculations are the fields of atomic physics, electronics, actuarial work, optics, and astronomy. The operator of the machine need not be a trained mathematician, but the problem must be coded by one.

R. H. O.

LIBRARY NOTES.

The Committee on Library desires to add to the collections any technical works that members would wish to contribute. Contributions will be gratefully acknowledged and placed in the library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat Service. Photostat prints of any material in the collections can be supplied on request. Orders received in the morning are filled the same day. The average cost for a print 9 × 14 inches is thirty-five cents.

The library and reading room are open on Mondays, Tuesdays, Wednesdays and Fridays from nine o'clock A.M. until five o'clock P.M., Thursdays from two until ten o'clock P.M., Saturdays from nine o'clock A.M. until twelve noon.

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NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION.

Sterilizing the Bacteriological Loop During Phenol Coefficient Determinations.—ROBERT K. JENNINGS AND C. H. LUDWIG. Rapid transfer of cultures at specified time intervals, as in phenol coefficient tests which require that a fresh transfer be made every thirty seconds, is greatly facilitated by the use of the device illustrated. When so short a time is allotted to each transfer there is some danger of skimping on the time required for sterilization of the wire or of not permitting it to cool sufficiently. The apparatus shown in use automatically regulates the time of heating and cooling the loop, while actually shortening the time required for this manipulation, as far as the operator is concerned.

FIG. 1. Loop sterilizer for use in making numerous rapid transfers.

Three standard loops in conventional holders are used. While one is in use, the previously used wire is being sterilized and the third is cooling. They are supported in a rack made of copper tubing mounted in discs of copper. The rack rotates on a rod adjustably attached to a wooden support. In the illustrated model this axle is attached to an arm fastened to the upright member, a procedure which permits adjustment of the position of the wires either forward and backward or from side to side. Once the rack has been so adjusted that only one wire is in the flame at a time it is not necessary to change it. If desired the adjustability could be sacrificed for the sake of simplicity and the rack supported directly from the baseboard.

A wing top burner is permanently attached to the upright wooden support. It is placed so that rotation of the rack brings one wire after another into the flame. Centering of the wires in the flame is

rendered automatic by a ratchet consisting of a round-headed bolt held against the bottom of the lower disc by a spring. Holes countersunk in the disc engage the head of the bolt, indicating how far the rack should be turned.

In use, a wire-holder is removed from the rack, the transfer is made, and the holder replaced in the same tube. The used wire is then swung into the flame and the wire which has previously been sterilized and cooled is taken for the next inoculation. The openings of the copper tubes are slightly flared to make insertion of the handles less difficult.

The wires are supported perpendicularly rather than horizontally since it was felt that insertion of the holders into horizontal tubes would be too awkward an operation. A horizontal model using spring clips in place of tubes to hold the instruments proved unsatisfactory. Vibration of the wire as the result of snapping the wire-holders into place had a tendency to shake loose drops of culture. The perpendicular arrangement has the further advantage of conserving space on the laboratory bench, an item of some importance when large numbers of transfers are to be made.

BOOK REVIEWS.

CAREERS IN SCIENCE, by Philip Pollack. 222 pages, illustrations, 14 × 21 cms. New York, E. P. Dutton & Company, 1945. Price $2.75.

For postwar careers Mr. Pollack foresees that science will offer as many opportunities as nearly any field. Considering each of four major divisions of science—chemistry, physics, biology and geology—he points out the various branches and industries which can employ scientists. To indicate but briefly the variety, there are for example plastics, synthetic rubber and textiles in chemistry; electronics, glass, meteorology and aviation in physics; medicine, tree surgery and sanitation in biology; and prospecting for minerals and petroleum in geology.

The scientist is offered three main types of work—the making of tests and maintaining of controls in already established processes and techniques, research looking to the betterment of those techniques or the development of entirely new processes or products, and teaching to enable coming generations to learn the techniques developed in the past. The author points out specific tasks that might fall to the beginning scientist and suggests as well various research problems that await solution.

Some of the traits necessary for a scientific career are indicated and mention is made of the not inconsequential matter of the pay envelope. Numerous examples are cited of individuals who have made notable contributions to science, several of whom are shown in the illustrations depicting scientists at work. A separate chapter surveys the opportunity for women in this expanding field.

With the increasing emphasis placed by both industry and government on the value of research, the number of positions in the field of science will unquestionably increase after the war. This volume will serve as an excellent survey for the young person interested in a scientific career.

G. E. PETTENGILL.

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Your Voice, by Douglas Stanley. 306 pages, illustrations, 16 × 24 cms., New York and Chicago, Pitman Publishing Corporation, 1945. Price $4.50.

Co-operative Electrical Research. 62 pages, illustrations, 19 × 25 cms. London, The British Electrical and Allied Industries Research Association, 1944.

Electrical Coils and Conductors, by Herbert Bristol Dwight. First Edition. 351 pages, illustrations, 14 × 22 cms. New York and London, McGraw-Hill Book Company, Inc., 1945. Price $5.00.

The Forest Industries Blaze New Trails. 32 pages, illustrations, 22 × 30 cms. Washington, D. C., Timber Engineering Company.

Fire Protection Through Modern Building Codes, by B. L. Wood. 244 pages, 15 × 24 cms. New York, American Iron and Steel Institute, 1945.

Making Patent Drawings, by Harry Radzinsky. 96 pages, drawings and illustrations, 19 × 26 cms. New York, The Macmillan Company, 1945. Price $3.00.

CURRENT TOPICS.

ARMY AND NAVY NOTES.

New Electronic Timer Gives Uniform X-Ray Exposures.—A photoelectric x-ray timing device now enables radiologists and technicians to obtain uniformly dense photofluorographic exposures with an overall increase in operating efficiency of 100 per cent. The electronic timer, developed by the Westinghouse Electric and Manufacturing Company, times each exposure properly, quickly, and automatically. The timer in effect, measures the light leaving the fluorescent screen on which the x-rays cast the object image. When sufficient light has left the screen for satisfactory film exposure, the photoelectric tube which scans the screen actuates a relay, opening the x-ray circuit and terminating the exposure.

Although first used in medical radiography for mass chest surveys on miniature roll films, the development promises to be generally useful and will include industrial x-ray analysis. Whether similar objects, such as castings, are moving rapidly on a conveyor or whether a variety of irregular objects must be x-rayed, the electronic timer will provide uniform exposures quickly and efficiently.

Use of the photoelectric cell to control exposures in photofluorography was first undertaken by Dr. Paul C. Hodges and Dr. Russel E. Morgan at the University of Chicago. The electronic timer takes advantage of the nature of the photoelectric tube whose output is proportional to the light striking the cathode.

The heart of the Westinghouse Phototimer is the so-called phototube camera consisting of a lens which scans the fluorescent screen, a photoelectric multiplier tube, and a condenser—thyratron—relay system. The phototube camera is mounted beneath the photofluorographic hood and "sees" the object image on the screen which is "seen" by the photographic camera at the apex of the hood. Light from the screen initiates a small current in the photoelectric tube which is greatly amplified. The output current charges the condenser until the proper ionization voltage is reached when the thyratron fires actuating the relay which opens the x-ray circuit and terminates the exposure. Proper choice and design of circuit constants enable the timer to maintain the correct exposure factor for uniformly dense film exposures.

Two factors account for the initial development of the timer for chest photofluorography: (1) the importance of mass chest surveys in controlling civilian tuberculosis and (2) the demands of the Armed Forces in examining large numbers of inducted men. "Miniature photofluorography—taking small-film photographs of full-scale images created on a fluoroscopic screen by the action of x-rays—is the only practical and economic way to conduct the mass chest surveys necessary to detect the presence and prevalence of tuberculosis in the whole population," explained Mr. A. P. Craig, manager of the X-Ray Division of the Westinghouse Company.

Until now, a major difficulty barring full utilization of the analytic capabilities of x-ray has been the lack of automatic exposure controls. The use of the

new phototimer, according to Henry D. Moreland, manager of engineering of the Westinghouse X-Ray Division, automatically assures a correct x-ray exposure, enabling the technician not only to double the number of exposures formerly made but to achieve better and more useful results as well.

"In order to evaluate accurately x-ray pictures," Mr. Moreland said, "it is important that the exposures be uniform in density and in what photographers call 'contrast'—the sharpness of difference between the light and the dark areas."

X-rayed objects may vary not only in size and thickness but even in the degree to which x-rays are transmitted. These variables make it difficult to obtain uniform film exposures. Expert technicians develop considerable skill in estimating x-ray penetration, but such evaluations and measurements are time-consuming and at best are only approximations. Therefore even the best technician can only attain comparative uniformity of film exposures. "The procedure is much like that used by a photographer who has no exposure meter," Mr. Moreland pointed out. "He must use his past experience to judge the proper setting of his camera to take into account light intensity, shutter speed, and diaphragm opening. The new Westinghouse Phototimer is a device to turn the speed and accurate perception of electronics to this task."

R. H. O.

Compressed Air Gives Flame Thrower Long Range.—(*Compressed Air Magazine*, Vol. 49, No. 12.) America's new M1-A1 flame thrower is a deadly weapon that can shoot around a corner almost at a right angle and has a maximum reach of 180 feet, as against 60 for the older type. It is a strange-looking gun with a crooked barrel connected by flexible tubing to three cylinders strapped on the soldier's back. The two outer ones contain the "ammunition," while the middle one is charged with compressed air that forces the fuel into the barrel and sends it flying like a tracer bullet toward its objective. At the muzzle is a burner to which hydrogen gas is fed from another smaller cylinder attached to the underside of the barrel. This gas is electrically ignited. The fuel tanks have to be refilled in the rear where the necessary safety precautions can be observed.

Two men are required to handle the thrower, one to carry it and another to operate the valves that regulate the fuel and the air supplies. It is fired from the hip and burns Gel Gas, a thickened gasoline that issues from the muzzle in a rod-like stream. As it travels along at high speed the outer surface is ablaze; but the core is consumed progressively and is still in a semisolid state when it strikes the target. Even at the maximum range, impact is so great as to cause the burning Gel Gas to spatter and to adhere to whatever it comes in contact with, quickly reducing everything combustible to ashes. The gun is usually fired in a series of 2-second bursts, as a sustained one would last only fifteen seconds. The complete equipment weighs 68 pounds and has been used with telling effect in the Japanese and European theaters of war.

R. H. O.

Abacá.—Abacá or Manila hemp—one of the war essentials cut off by the Japanese advance into the Pacific—is a crop that has responded rapidly to the urgent effort of the United States to make certain of a supply by stimulating

production in the Western Hemisphere. There will not be as much abacá available in 1945 as is needed, but reports to the Office of Foreign Agricultural Relations of the U. S. Department of Agriculture indicate that in the fourth year after Pearl Harbor the Latin American growers will be able to produce a substantial supply.

Abacá is a "middle-distance" crop in point of growing time. It is not an annual with the rapid and quick multiplication and yield possible with the cereal grains. But it is not like some of the fruits and forest products in which a tree must grow for years before it yields—some of the nut crops and rubber, for example.

In the Philippines, abacá was not usually harvested until the plant was about three years old. The Central American growers, coached by American plantsmen, have been doing better. Giving the plants better care, they have advanced the harvest period to less than two years after planting. Fortunately an American controlled tropical fruit company had been experimenting with abacá growing for several years. It is a crop that is suited to some of the Central American lands where banana growing has had to be abandoned because of the spread of banana disease caused by a fungus. This abacá acreage taken from banana plantations provided seed for the greatly expanded acreage of abacá planted in Panama, Guatemala, Costa Rica, and Honduras which is coming into bearing in time to help replenish the shrinking stockpiles of this important fiber that is greatly needed to supply the Navy with ropes and cordage.

R. H. O.

Staypak—Improved Wood Without Resins.—Removal of military restrictions on certain information has enabled the U. S. Forest Products Laboratory at Madison Wis. to announce another Laboratory improved wood—staypak. This is a heat-stabilized high-density product made by compressing either solid wood or many layers of thin veneers. The material contains no resin except, in the case of the laminated product, normal amounts of resin adhesive to bond the veneers during pressing.

Staypak is an outgrowth of the metal shortage encountered in early stages of the war production program. Under the stimulus of military demands for modified woods to serve primarily in aircraft production, the Laboratory developed compreg—modified wood without normal wood's tendency to swell and shrink, much harder than normal wood, moisture-resistant, and also notably resistant to chemicals, insects and decay. Compreg however, because it is a combination of wood and synthetic resin, exhibits some of the characteristics of the contained resin. Although most of its strength properties are high, it is more brittle than the original wood.

Because of this characteristic, the need for a compressed, stabilized wood without the embrittling resin, was recognized. Dr. A. J. Stamm and his associates of the Forest Products Laboratory were familiar with improved woods made simply by compression. They also knew that these compressed woods in general were susceptible, under high moisture conditions, not only to ordinary swelling, but also to "springback," a release of pent-up stresses resulting from compression, and a return to original unpressed dimensions. To them this was an understandable phenomenon. The cells of the wood, deformed during

the pressing, tend to resume their original stress-free shape. On swelling, the release of pent-up stresses is started, and because swelling softens or plasticizes the wood, further release of the stresses is spontaneous.

The obvious solution of the problem lay in avoiding the stresses by some form of plasticizing softening action while the wood was still under pressure. The scientists reasoned that the stresses could be prevented if the lignin cementing material between the fibers could be made to flow somewhat during the pressing process. Lignin can be made to flow by applying heat alone, but this requires such high temperatures that the wood tends to burn. The use of plasticizers to lower the flow temperature of lignin was tried. Fortunately, water proved to be the best plasticizer.

Finally it was determined that all that was necessary to produce a compressed product practically free from springback was to compress for the proper length of time at somewhat higher temperatures and moisture contents than had previously been used. The highly stable properties are developed only when the wood is virtually completely compressed to a specific gravity between 1.3 and 1.4.

The cost of staypak is roughly proportional to the increase in specific gravity as compared with uncompressed wood. The present ceiling price for 1-inch-thick Douglas fir plywood of the best grade is just under 19 cents per square foot at the mill. Douglas fir laminated staypak made from select veneer should cost in the neighborhood of 65 cents per board foot to manufacture, in thickness not exceeding 1 inch; laminated birch staypak would run about 90 cents per board foot on the same basis as against 40 cents per square foot for birch plywood 1 inch thick. Staypak made from solid wood would, of course, be less expensive.

The production of staypak calls for special equipment. If the laminated product is to be made, essentials include a glue spreader and a conditioning chamber for bringing the veneers to proper moisture content for pressing. More critical in view of existing priorities and shortages affecting pressing equipment is the need for hot presses with platens that can be water-cooled and that are capable of exerting a pressure of 2,000 pounds per square inch over the full platen area. If the platens are steam heated, a steam pressure of 120 pounds per square inch will be required.

Patents on the staypak process have been applied for by the Forest Products Laboratory. If patents are granted, the process will be licensed to interested manufacturers by the Secretary of Agriculture.

Details of manufacturing conditions of staypak are contained in a report (No. 1580) available on request at the U. S. Forest Products Laboratory, Madison, Wis. Photographs illustrating this release also are available to publications which can use them.

R. H. O.

Research and Progress in Steel Manufacture.—Important advances in both fundamental research and in relating research findings to specific improvements in steel products were made in laboratories and in the plants of subsidiary companies of United States Steel Corporation during 1944. While many of the studies concerned steel for war uses and the results of these investigations must remain secret temporarily, numerous other results are of a general nature which will be of value to the entire steel industry, as well as to the users of steel.

Early in the year Corporation researchers issued an Atlas of Isothermal Transformation Diagrams greatly extending and illustrating graphically the fundamental principles of steel heat treatment. This publication was widely acclaimed as a valuable contribution not only to the war effort but to the general advancement of the art and science of heat treatment. Additional work on the mechanical properties, hardenability and isothermal transformation diagrams of numerous National Emergency steels has now been completed and studies are being continued further on heat treatment for specific needs.

New data is expected shortly in connection with the development of high grade tensile steels, and a vast amount of effort has been devoted to forging and machining. The results of these studies remain restricted, but are expected to be made available in the near future to the manufacturers of steel products.

Researches have been continued on the use of intensifiers, such as boron, and on hardenability. Work on electrolytic tin plate and other coatings for steel sheet, strip and wire has made great strides. New pickling practices have been perfected, and some of the perplexing problems relating to waste pickle liquor have advanced toward solution.

Much practical research on deep drawing was performed in connection with the steel cartridge case program and these results will be of value to civilian industry when steel again becomes available for peace-time uses. Advances have also been made in the manufacture of steel sheet for enameling.

Spectrographic equipment is now in standard use in several corporation plants as a result of recent development work, and advances have been made during the past year in the further application of pyrometry to open hearth, electric furnace and Bessemer practice and control. Much valuable work has been done also in developing standards and improved practices for various grades of Bessemer steel. As a direct result of this research, a considerable amount of Bessemer seamless tubing is now being made. Perfected dephosphorizing methods have increased the utility of some Bessemer grades and an increasing quantity of alloy steel is being made by the Bessemer process.

Standards and practices relating to NE and other alloy steels have been improved. Many steels now being used exclusively for war purposes have established a permanent place for themselves in industry as a result of recent technologic developments within the mills of United States Steel Corporation subsidiaries.

Research on fundamentals, for which the research laboratory of United States Steel Corporation at Kearny, N. J., is particularly well equipped, has been continued on a wide variety of problems and has been extended into many of the new laboratories in the subsidiary company mills. The results of these investigations are applicable not only to immediate war problems, but also to post-war products.

A satisfactory magnetic method was developed at the Kearny laboratory for inspecting steel fabricated into gas cylinders for pneumatic life rafts. Much special work on magnetic testing of steels for various purposes was also completed there and more is in progress. High temperature tests, including time-consuming creep strength studies, were continued through the year.

Among outstanding recent research achievements of this laboratory is the development and perfection of special equipment for testing the so-called "microhardness" of metals, which enables the research workers to determine

with precision the hardness of individual grains or constituents of steel so small that they are visible only at high magnification under a microscope.

Various advances were made in operating practice and equipment during 1944. These contributed substantially to the continuation of the exceptionally high operating rate which has been maintained in coke, blast furnace, and open hearth operations and made it possible for open hearth operators to carry their full share of the burden imposed by the continuing demand for alloy steel. Methods were perfected to facilitate the use of alloy scrap and alloy steel turnings by United States Steel plants, and Pittsburgh district open hearth plants increased their use of nodulized ore as produced by a Corporation plant. Additional hot-topping facilities increased the production of ordnance steels.

Much progress has been made by Steel Corporation subsidiaries in improving ingot heating facilities and in the control of rolling temperatures so as to improve the quality of the finished product. New, high-speed rod mills were brought into successful operation at the Joliet, Ill., and Pittsburg, Calif., mills. Numerous other improvements have been made throughout all subsidiary company mills to make possible a continued high production rate while maintaining high standards of quality.

R. H. O.

A Cinderella Metal.—(*Compressed Air Magazine*, Vol. 49, No. 11.) Only a few years ago, magnesium was a comparatively rare metal, rather high-priced and more or less of a laboratory material used principally in flashlight powder, fireworks, and flares. Scientists spoke of its admirable qualifications for various services where light weight was a requisite and predicted that it would some day attain considerable industrial importance. But no one could forecast the phenomenal rise to prominence that has been its lot as one of the Cinderella metals of the war.

A 1921 edition of an American text-book on chemistry stated that "magnesium is of comparatively little commercial importance," and it continued to be regarded in that light during the ensuing years. The Germans, however, didn't share that viewpoint. When Nazi planes were shot down in England early in the conflict, some of them were surprisingly found to contain up to 1000 pounds of magnesium parts that had been fabricated from forgings, castings, extrusions, and rolled material.

As the United States neared involvement in the war, we had virtually no domestic sources of supply. We got what we urgently needed from abroad and, meanwhile, began feverishly to prepare to exploit our own resources. A revolutionary process for extracting magnesium from sea water was successfully applied on the Gulf Coast, and several establishments were set up for recovering the metal from its ores. Jointly, they gave us magnesium in continually increasing amounts until our war needs were fully met.

Now an anomaly arises, and the metal promises to become temporarily too abundant. On October 5 the War Production Board terminated its control of magnesium by removing the ban on its use for civilians. By the stroke of a pen its career as a critical metal was ended. The restriction was lifted as a result of the curtailment in military-aircraft production, and the decrease in the demand for magnesium that followed is but a foretaste of what may be ex-

pected when hostilities cease. It is predicted that the fabricated metal will then drop from 15 million pounds a month to 1½ million.

The Magnesium Association has already moved to take up the slack, and the indications are that the public will be the real beneficiary of the wartime research and production program. The major problems of making and fabricating magnesium and its alloys have been solved, and a complete range of products will be available to meet the varying needs of peacetime trade.

Magnesium is two-thirds as heavy as aluminum and only one-fourth as heavy as steel. This relative lightness counts heavily when it is translated into the economics of the aviation industry, since every pound saved in constructing a transport plane will return around $400 through increased carrying capacity during the life of the craft. Along with light weight, some magnesium alloys have a tensile strength comparable to steel, and they can be machined three times faster. Although always found in combination with other elements, magnesium is very abundant in Nature, constituting 2 per cent. of the earth's crust and 0.13 per cent. of sea water.

Even before the war began, one manufacturer of printing presses had been able to increase the speed of his presses 25 per cent. by employing this lightest of all structural metals. With this example before them, magnesium producers are looking towards its use in numerous other mechanisms and appliances such as office and knitting machines, hand tools, radios, and cameras. They also visualize it as a competitor of some plastics. Favorable to magnesium are its ready machinability and current low price of around 20 cents a pound. Many engineers now in the aviation industry will, when they shift back to peacetime manufacturing plants, be "magnesium conscious" and lean towards it wherever it can be applied. The magnesium industry is already planning an intensive research program with facilities for passing on to those who can make use of it any information that may be gained.

R. H. O.

An Odorless Household Fly Spray.—Chemists in the Agricultural Research Administration, U. S. Department of Agriculture, have perfected an odorless and nonirritating household fly spray.

This development is the result of a new method of purifying the powerful insecticide, pyrethrum, so that it will be less likely to produce irritating rashes or hay fever symptoms, which have limited its use for certain purposes in the past. With this discovery scientists at the Beltsville Research Center, Beltsville, Md., say pyrethrum sprays, nontoxic to man but deadly to flies, mosquitoes, and other household pests, can now be used without discomfort in the home or any place where people congregate.

The odorless spray was developed by three chemists in the Bureau of Entomology and Plant Quarantine, W. F. Barthel, H. L. Haller, and F. B. LaForge. They discovered a new and simple method by which a new solvent called nitromethane can be used to produce practically pure pyrethrins from the impure petroleum extract of pyrethrum flowers. Scientists say that the new solvent takes out practically all of the pyrethrins, the toxic ingredient in these insecticides, but leaves most of the irritating impurities in the petroleum solvent.

The nitromethane solution is then passed through charcoal which operates as in World War I gas masks to remove any remaining irritating material. The nitromethane is then distilled off, leaving practically pure (90–100 per cent.) pyrethrins, with no irritating residues.

Used in the new automatic spraying device called the aerosol bomb, a Department of Agriculture wartime invention provided to the armed forces for wartime control of mosquitoes, entomologists say this aerosol promises to be an effective postwar weapon against flies in homes, restaurants, airplanes and public places, and to kill other household pests such as cockroaches, bedbugs, ants, mosquitoes, house spiders, silverfish, chiggers, carpet beetle larvae, dog ticks, and dog fleas.

The aerosol sprayer is a small handy can dispenser that holds a liquefied gas such as Freon, which is now used in household refrigerators. When a valve is opened this aerosol dispenser emits a foglike spray of poisonous droplets so tiny that they kill more insects than ordinary coarse sprays. While not yet available for civilian sale, more than 13 million of these Freon-aerosol "bombs" have been supplied to the armed forces for use against disease-carrying insects. As a result of such extensive use, this kind of aerosol spray dispenser has been thoroughly tested and will be available for household use as soon as a commercial product can be manufactured for the home market after the war ends.

The new pyrethrum process not only produces a much more concentrated and powerful insect poison than has been obtained heretofore, but Department of Agriculture chemists report, the resulting purified pyrethrins are also ideal for use in the Freon-aerosol bombs.

R. H. O.

Fiberglas-Resin Sheets Protect Plane Fuselage.—Glass in the form of fine fibers, twisted and plied into yarns and woven into cloth, is now being employed by Pennsylvania Central Airlines to protect the metal skins of the fuselages of its planes.

Cargo spaces of transport planes turned back to Pennsylvania Central Airlines by the Army, and now being reconditioned for civilian use, are being lined with thin, strong sheets consisting of layers of Fiberglas cloth bonded and impregnated with a contact-pressure, thermo-setting MR-type resin. The lining guards the fuselage skin against contact with heavy items of cargo, during loading and unloading.

The Fiberglas-resin sheets, one-sixteenth of an inch thick, do not expand or contract under the influence of temperature and humidity changes such as are encountered by planes flying at varying heights. They have greater impact strength, and are lighter in weight, than the aluminum sheathing formerly used to line the cargo spaces.

The finished sheets are manufactured by the Plastics Division of Continental Can Company. The Fiberglas cloth, which provides the strength element, is manufactured by Owens-Corning Fiberglas Corporation.

R. H. O.

Cotton Clothing For Warmth.—Development of special military fabrics is likely to benefit civilians after the war, says a special report on postwar marketing of cotton prepared by an inter-bureau committee of the U. S. Department of Agriculture.

The report refers particularly to light-weight, wind-resistant, and water-repellant cotton fabrics that have been developed by the Quartermaster Corps. The committee says that tests by the Army show that light-weight, tightly-woven cotton fabrics, when treated with a chemical finish, provide warmth without the cumbersome weight of woolen fabrics. It is expected that in the post-war period, when manufacturers can supply such fabrics to civilian buyers, they will be used in quantity for all types of overclothing, and particularly for winter work clothing.

R. H. O.

Navy-Developed "Live" Blood Portable Refrigerators a Big Factor in Saving Lives.—An indication of how the modern science of refrigeration is meeting the demands of war is afforded by the performance of the portable refrigerators developed by the Navy for air transportation to the most distant war fronts of "live" human blood collected by the Red Cross.

Live blood is blood as it comes from the donor's veins. In some cases, even though blood plasma has saved a wounded man during the initial shock period, subsequent heavy loss of blood has caused death because there wasn't enough blood left to carry needed oxygen.

Transfusions of live, or whole, blood are required to meet such situations, but until recently it was not possible to keep live blood for more than a week, making transportation to distant war fronts impracticable, even with planes available to carry it.

Two developments have completely changed the picture. It was found that if a solution of citric acid, sodium citrate, and dextrose was added to the live blood, it would retain 75 per cent. of its original value for as long as 21 days. The Navy developed a light, portable refrigerator that would keep the blood at a 40 to 50 degree temperature for over 60 hours at a stretch.

The refrigerator consists of a plywood case lined with Fiberglas insulation two inches thick. In addition to providing an efficient barrier to the passage of heat, the glass fibers are extremely light in weight and do not pack down under shock or vibration, leaving gaps for heat to pass through.

Glass jars of live blood are placed in racks surrounding a metal container filled with ice. Weight of the refrigerator and the ice barely exceeds the weight of the blood. The whole "package" is light enough to be handled by one man.

Transport planes are now flying the refrigerators and their contents of live blood on a daily schedule from San Francisco to Pearl Harbor, and thence to island bases across the Pacific. The program is under Navy jurisdiction, but the blood is made available to Army as well as Navy medical officers.

Live blood complements blood plasma, but in no way lessens the need for the latter. Only blood from persons with "O," or "general," type blood can be used as live blood in the present program. Donors are tested by the Red Cross centers participating in the live blood program to determine whether they have the needed type of blood. The blood of type "O" donors is flown overseas, while that of other donors is processed into plasma.

R. H. O.